It’s ok to work together, but don’t copy without comprehension. The symbol (E) means that at least part of this problem is a former or potential exam question.

Ungraded. Fraleigh § IV.20 (p.189)– 5,11,13; § IV.22 (p.182)– 1, 3.

1a. Fraleigh § IV.18 (p.176) – 44.
1b. Fraleigh § IV.20 (p.189)– 12, 14 .

2. (E) Suppose φ : Z/4Z → Z/6Z is a homomorphism and φ([1]4) = [a]6. Determine all possible values for a, and determine φ([2]4) and φ([3]4) in these cases.

3. (E) Let g(x) = x^2 + x. Show that 0, 2, 3, and 5 are roots of g in Z/6Z[x], and find two different ways to write g(x) = (x + a)(x + b) in this ring. (Hint: one of them is pretty easy.)

4. (E) Find, with explanation, the last two digits of 2^{63} and 3^{63} in the usual base 10 expression. That is, find m, n, 0 ≤ m, n ≤ 99, such that 2^{63} ≡ m mod 100 and 3^{63} ≡ n mod 100. (It’s easy enough to do the computation by computer; I’m interested more in the explanation than the answer.)

5. (E) Fraleigh § IV.22 (p.208) – 27. Remark: This is not a hard problem, but it is a good test of vocabulary. Observe that, if D is a ring homomorphism then D(fg) = D(f)D(g). Try a few examples to see if this works.

6. (E) Solve the equation x^2 ≡ 1 mod m for m = 4,8,16. Use the answers to make a conjecture about the solution to x^2 ≡ 1 mod 2^r, r ≥ 3.
(extra-credit, 1/4 point) Solve the equation for m = 32 and prove your conjecture.