Math 417 **Homework 9** **Due Friday, April 12, 2019**

It's ok to work together, but don't copy without comprehension. The symbol $(\mathcal{E})$ means that at least part of this problem is a former or potential exam question.

Ungraded. *Fraleigh* § IV.20 (p.189)– 5,11,13; § IV.22 (p.182)– 1, 3.

1a. *Fraleigh* § IV.18 (p.176) – 44.

1b. *Fraleigh* § IV.20 (p.189)– 12, 14 .

2. $(\mathcal{E})$ Suppose $\phi : \mathbb{Z}/4\mathbb{Z} \mapsto \mathbb{Z}/6\mathbb{Z}$ is a homomorphism and $\phi([1]_4) = [a]_6$. Determine all possible values for a, and determine $\phi([2]_4)$ and $\phi([3]_4)$ in these cases.

3. $(\mathcal{E})$ Let $g(x) = x^2 + x$. Show that 0, 2, 3, and 5 are roots of g in $\mathbb{Z}/6\mathbb{Z}[x]$, and find **two** different ways to write $g(x) = (x + a)(x + b)$ in this ring. (Hint: one of them is pretty easy.)

4. $(\mathcal{E})$ Find, **with explanation**, the last two digits of 2^{63} and 3^{63} in the usual base 10 expression. That is, find $m, n,\ 0 \le m, n \le 99$, such that $2^{63} \equiv m \mod 100$ and $3^{63} \equiv n \mod 100$. (It's easy enough to do the computation by computer; I'm interested more in the explanation than the answer.)

5. $(\mathcal{E})$ *Fraleigh* § IV.22 (p.208) – 27. Remark: This is not a hard problem, but it is a good test of vocabulary. Observe that, if D is a ring homomorphism then $D(fg) = D(f)D(g)$. Try a few examples to see if this works.

6. $(\mathcal{E})$ Solve the equation $x^2 \equiv 1 \mod m$ for $m = 4, 8, 16$. Use the answers to make a conjecture about the solution to $x^2 \equiv 1 \mod 2^r,\ r \ge 3$.

(extra-credit, $\frac{1}{4}$ point) Solve the equation for $m = 32$ and prove your conjecture.