It's ok to work together, but don’t copy without comprehension. The symbol (E) means that at least part of this problem is a former or potential exam question.


1a. Fraleigh §IV.18 (p.175) – 23.
1b. Fraleigh §IV.18 (p.176) – 46 (Hint: the binomial theorem).

2. Fraleigh §IV.19 (p.182)– 12. (Hint: \((a + b)^9 = ((a + b)^3)^3\).)

3a. (E) Determine the zero-divisors in \(R = \mathbb{Z}/0\mathbb{Z}\). If \(x\) is a zero divisor, find a non-zero \(y\) so that \(xy = 0\).
3b. (E) Determine the units in \(R = \mathbb{Z}/10\mathbb{Z}\). If \(x\) is a unit, find \(y\) so that \(xy = 1_R\).

4. (E) Let \(R = \mathbb{Z}/12\mathbb{Z}\). Compute the seven polynomials
\[x^2, (x + 1)^2, (x + 2)^2, (x + 3)^2, (x + 4)^2, (x + 5)^2, (x + 6)^2\]
in \(R[x]\). Reminder: the coefficients of your answers must lie in \(R\); to be precise, \((x + 3)^2\) really means \(([1]_{12}x + [3]_{12})^2\), etc.

5 & 6. (E) Consider the ring \(R = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}\), whose elements are \(([a]_2, [b]_4)\), where \(a \in \{0, 1\}, b \in \{0, 1, 2, 3\}\). Recall that we have already studied objects like \(R\) as a group, under the operation “+”.

a. Write down the addition table for \(R\) and identify \(0_R\). (There are 8 elements in \(R\).)
b. Write down the multiplication table for \(R\) and identify \(1_R\). (There are 8 elements in \(R\).
c. Determine the zero-divisors of \(R\), and for each zero divisor \(x = ([a]_2, [b]_4)\), find a non-zero \(y = ([c]_2, [d]_4)\) so that \(xy = 0_R\).
d. Determine the units of \(R\), and for each unit \(x = ([a]_2, [b]_4)\), find \(y = ([c]_2, [d]_4)\) so that \(xy = 1_R\).