Math 417 **Homework 8** **Due Friday, April 5, 2019**

It's ok to work together, but don't copy without comprehension. The symbol ($\mathcal{E}$) means that at least part of this problem is a former or potential exam question.

Ungraded. *Fraleigh* §IV.18 (p.174, 175)– 3, 5, 15; §IV.19 (p.182)– 3, 4.

1a. *Fraleigh* §IV.18 (p.175) – 23.

1b. *Fraleigh* §IV.18 (p.176) – 46 (Hint: the binomial theorem).

2. *Fraleigh* §IV.19 (p.182)– 12. (Hint: $(a+b)^9 = ((a+b)^3)^3$.)

3a. ($\mathcal{E}$) Determine the zero-divisors in $R = \mathbb{Z}/0\mathbb{Z}$. If x is a zero divisor, find a non-zero y so that $xy = 0$.
3b. ($\mathcal{E}$) Determine the units in $R = \mathbb{Z}/10\mathbb{Z}$. If x is a unit, find y so that $xy = 1_R$.

4. ($\mathcal{E}$) Let $R = \mathbb{Z}/12\mathbb{Z}$. Compute the seven polynomials

$$x^2, (x+1)^2, (x+2)^2, (x+3)^2, (x+4)^2, (x+5)^2, (x+6)^2$$

in $R[x]$. Reminder: the coefficients of your answers must lie in R; to be precise, $(x+3)^2$ really means $([1]_{12}x + [3]_{12})^2$, etc.

5 & 6. ($\mathcal{E}$) Consider the ring $R = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$, whose elements are $([a]_2, [b]_4)$, where $a \in \{0,1\}, b \in \{0,1,2,3\}$. Recall that we have already studied objects like R as a group, under the operation "+".

a. Write down the addition table for R and identify 0_R. (There are 8 elements in R.)

b. Write down the multiplication table for R and identify 1_R. (There are 8 elements in R.)

c. Determine the zero-divisors of R, and for each zero divisor $x = ([a]_2, [b]_4)$, find a non-zero $y = ([c]_2, [d]_4)$ so that $xy = 0_R$.

d. Determine the units of R, and for each unit $x = ([a]_2, [b]_4)$, find $y = ([c]_2, [d]_4)$ so that $xy = 1_R$.