Math 417 **Homework 7.1 (sigh)** **Due Friday, March 15, 2019**

It's ok to work together, but don't copy without comprehension. The symbol ($\mathcal{E}$) means that at least part of this problem is a former or potential exam question.

Ungraded. *Fraleigh* ; §1.14 (p.142) – 3,5,9.

1a. *Fraleigh* §1.13 (p.134) – 22. The operation in the groups $\mathbb{Z}$ and $\mathbb{Z} \times \mathbb{Z}$ is addition.

1b. *Fraleigh* §1.14 (p.142) – 2. In class notation, the group would be written as

$$(\mathbb{Z}/4\mathbb{Z}) \times (\mathbb{Z}/12\mathbb{Z})/(\langle [2]_4 \rangle \times \langle [2]_{12} \rangle.$$

2. ($\mathcal{E}$) Suppose G is a group and $a \in G$ is any element. Define $\Phi_a(x) = a^{-1}xa$. Prove carefully that Φ_a is an automorphism of G; that is, an isomorphism of G. Reminder (not really a hint). You have to prove each of the following: that Φ_a is one-to-one and onto and respects the operation.

3. ($\mathcal{E}$) Let $G = \mathbb{Z}/16\mathbb{Z} \times \mathbb{Z}/42\mathbb{Z}$. **16, not 18. Sorry.**

3a. Find an element $a \in G$ whose order is equal to 28.

3b. Prove that there is no element $b \in G$ whose order is equal to 5.

4. ($\mathcal{E}$) Find all homomorphisms ϕ from the Klein group V to the group $G = (\mathbb{Z}/7\mathbb{Z}, +)$. Hint: let $V = \{e, x, y, z\}$ and suppose $\phi(x) = [a]_7$. What possible values can a take?

5 & 6. ($\mathcal{E}$) Let $G = (\mathbb{Z}/6\mathbb{Z}, +)$ and $H = C_{12} = \langle a \rangle = \{e, a, \ldots, a^{11}\}$. Define a homomorphism $\phi : G \to H$ by $\phi([1]_6) = a^4$.

a. Determine K, the kernel of ϕ, as a subgroup of G (Hint: you will want to compute $\phi([j]_6)$ for all the elements $[j]_6 \in G$.)

b. Determine the image of ϕ as a subgroup of H.

c. Determine the factor group G/K. By this I mean: write down the elements of the group G/K and its multiplication table.