Math 417 **Homework 6.2** **Due Friday, March 8, 2019**

It's ok to work together, but don't copy without comprehension. The symbol ($\mathcal{E}$) means that at least part of this problem is a former or potential exam question.

Ungraded. *Fraleigh* §1.11 (p.110)– 1,3,5; §1.13 (p.133,134) – 17, 19.

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1. *Fraleigh* §1.10 (p.103) – 37. Hint: For any element $a \in G$, $\langle a \rangle$ is a subgroup in G. If G has no proper subgroups, then $\langle a \rangle$ must either be $\{e\}$ or all of G.

2.. *Fraleigh* §1.11 (p.110) – 2, 4.

3a. Test 1, revisited, so ($\mathcal{E}$).
Prove that $G = ((\mathbb{Z}/11\mathbb{Z})^*, \times)$ is a cyclic group of order 10. (Hint: take powers of $[n]_{11}$ for one particular well-chosen n; you don't need to write out the whole multiplication table.)
3b. Find *all* $[a]_{11}$ for which $\langle [a]_{11} \rangle = G$. (If $[n]_{11}$ is a generator of G, write $a = n^k$. We have a theorem which says something about k and 10.)
3c. Find it two different isomorphisms: Φ_1, Φ_2 from C_{10} to G. Write out the entire set of values: $\Phi_j([0]_{10}), \Phi_j([1]_{10}), \Phi_j([2]_{10}), \ldots$ for your choices of Φ_j.

4. Test 1, revisited, so ($\mathcal{E}$). This is an improvement of the version given in Homework 6 (first version)

Look at the group you defined on HW 2 #2, where $S = \mathbb{R} \setminus \{1\}$, that is, the set of all real numbers except 1, and the binary operation of S is defined by by

$$a \circ b = a + b - ab,$$

You have already proved that $(S, \circ)$ is a group. (I forgot to prove closure, but did so in class.) You already know what the identity is and and inverse formula you found in that problem. You don't have to prove those again.

Determine, with proof, the group generated by 3 with respect to $\circ$. Don't forget the inverse of 3. Play around and take powers: there's a nice pattern that shows up. Prove it! (Hint: the group is infinite.)

5. ($\mathcal{E}$) 6. Suppose p and q are different primes, and $G = C_p = \langle a \rangle$ and $H = C_q = \langle b \rangle$ and suppose ϕ is a homomorphism from G to H. Prove that ϕ is trivial; that is, $\phi(g) = e_H$ for every $g \in G$. Hint: pick an interesting $g \in G$ and suppose $\phi(g) = b^k$. What can you say about k?

6a. ($\mathcal{E}$) Suppose $H = \{e, a, b\}$ is a normal subgroup of a group G. Suppose $g \in G$ has the property that $ga \neq ag$, Prove that $ga = bg$. (Hint: don't panic!)
6b. ($\mathcal{E}$) Suppose H is a subgroup of G and $H = \{e, a\}$ is a subgroup of G of order 2. Prove that H is a normal subgroup of G if and only if $ag = ga$ for every $g \in G$. (Hints: write down what the left and right cosets of H look like, and don't panic!)