Math 417 **Homework 6** **Due Friday, March 8, 2019**

It's ok to work together, but don't copy without comprehension. The symbol ($\mathcal{E}$) means that at least part of this problem is a former or potential exam question.

Ungraded. *Fraleigh* §1.11 (p.110)– 1,3,5; §1.13 (p.133,134) – 17, 19.

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1. *Fraleigh* §1.10 (p.103) – 37. Hint: For any element $a \in G$, $\langle a \rangle$ is a subgroup in G. If G has no proper subgroups, then $\langle a \rangle$ must either be $\{e\}$ or all of G.

2.. *Fraleigh* §1.11 (p.110) – 2, 4.

3a. Test 1, revisited, so ($\mathcal{E}$).
Prove that $G = ((\mathbb{Z}/11\mathbb{Z})^*, \times)$ is a cyclic group of order 10. (Hint: take powers of $[n]_{11}$ for one particular well-chosen n; you don't need to write out the whole multiplication table.)
3b. Find *all* $[a]_{11}$ for which $\langle [a]_{11} \rangle = G$. (If $[n]_{11}$ is a generator of G, write $a = n^k$. We have a theorem which says something about k and 10.)
3c. Find it two different isomorphisms: Φ_1, Φ_2 from C_{10} to G. Write out the entire set of values: $\Phi_j([0]_{10}), \Phi_j([1]_{10}), \Phi_j([2]_{10}), \ldots$ for your choices of Φ_j.

4. Test 1, revisited, so ($\mathcal{E}$).
Recall the group defined in *Fraleigh* §1.4 − 19, p.46, which appeared on Homework 2. Recall also the identity and inverse formula you found in that problem. Determine, with proof, the group generated by 2 with respect to "*". You should include the inverse of 2. Play around: there's a nice pattern.

5. ($\mathcal{E}$) 6. Suppose p and q are different primes, and $G = C_p = \langle a \rangle$ and $H = C_q = \langle b \rangle$ and suppose ϕ is a homomorphism from G to H. Prove that ϕ is trivial; that is, $\phi(g) = e_H$ for every $g \in G$. Hint: pick an interesting $g \in G$ and suppose $\phi(g) = b^k$. What can you say about k?

6a. ($\mathcal{E}$) Suppose $H = \{e, a, b\}$ is a normal subgroup of a group G. Suppose $g \in G$ has the property that $ga \neq ag$, Prove that $ga = bg$. (Hint: don't panic!)
6b. ($\mathcal{E}$) Suppose H is a subgroup of G and $H = \{e, a\}$ is a subgroup of G of order 2. Prove that H is a normal subgroup of G if and only if $ag = ga$ for every $g \in G$. (Hints: write down what the left and right cosets of H look like, and don't panic!)