Math 417	**Homework 5**	**Due Friday, March 1, 2019**

It's ok to work together, but don't copy without comprehension. The symbol ($\mathcal{E}$) means that at least part of this problem is a former exam question.

Ungraded. *Fraleigh* §1.9 (p.94) – 9, 11; §1.10 (p.101) – 1, 3

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1. *Fraleigh* §1.8 (p.85) 40. This particular question is simpler than the full setting of the problem, so I'll restate it. Suppose A is a finite set and $b \in A$ is chosen. Let S_A denote the group of all permutations of A, and let $G = \{\sigma \in S_A \ : \ \sigma(b) = b\}$. Is G a subgroup of S_A?

2.. *Fraleigh* §1.9 (p.96) 34. One example is not a proof! Here's a way to start: let $\sigma = (x_1 x_2 \cdots x_{2n+1})$ be a cycle of odd length. What is σ^2?

3. ($\mathcal{E}$) Suppose G is a group and $\Phi(x) = x^{-1}$ is an automorphism of G (that is Φ is an isomorphism from G to G.) Prove that G is abelian. Hint: evaluate $(xy)^{-1}$ in two different ways.

4. ($\mathcal{E}$) Let $G = D_4$ be the dihedral group with 8 elements. Recall that the elements of D_4 are:

$$\{(1)(2)(3)(4), (1234), (13)(24), (1423), (12)(34), (14)(23), (1)(24)(3), (13)(2)(4)\}$$

Determine the left and right cosets of the subgroup $H_1 = \{(1)(2)(3)(4), (12)(34)\}$.

5. ($\mathcal{E}$) Suppose G is an *abelian* group and let H denote the set of squares in G. That is,

$$H = \{g^2 : g \in G\} = \{g * g : g \in G\}.$$

Prove carefully that H is a subgroup of G. You will need to use the fact that multiplication in G is commutative! (Don't worry about deciding whether H is a proper subgroup.)

6. ($\mathcal{E}$) Suppose $H = \{h_1, \ldots, h_m\}$ is a subgroup of G, a finite group. Let

$$K = \{a \in G \ : aH = Ha\};$$

That is,

$$a \in K \iff \{ah_1, \ldots, ah_m\} = \{h_1 a, \ldots, h_m a\}$$

as **sets**.

6a. Prove that $H \subseteq K$.

6b. Prove that K is a subgroup of G.

Please note that you cannot assume that (i) G is abelian or (ii) $ah = ha$ for all elements $h \in H$. In the definition, you are only told that aH and Ha are equal **as sets**.