Math 417 **Homework 4** **Due Friday, February 15, 2019**

It's ok to work together, but don't copy without comprehension. The symbol ($\mathcal{E}$) means that at least part of this problem is a former exam question. The order of a permutation (see problems 5 and 6) is a topic that will be on Test 2, not Test 1.

Ungraded. The answers to these are in the back, but I will write full solutions and you are "responsible" for them: *Fraleigh* §1.6 (p.67) – 46 Hint: observer that if $n \geq 1$, then $a(ba)^{n-1}b = (ab)^n$ by associativity; §1.8 (p.83) – 1,3.

1a. *Fraleigh* §1.5 (p.58) – 43. Use Theorem 5.14.

1b. *Fraleigh* §1.8 (p.83) – 2.

2 ($\mathcal{E}$) Suppose G is a group and a and b are elements. Let $H = \langle a \rangle$ and $K = \langle b \rangle$ be the cyclic subgroups of G generated by a and b. (You cannot assume that G is a cyclic group!) Suppose $a \in K = \langle b \rangle$ and $b \in H = \langle a \rangle$.

Prove that $H = K$. General advice: one good way to show that two sets S and T are equal is to show that $S \subseteq T$ and $T \subseteq S$.

3. ($\mathcal{E}$) Suppose G is a group and H and K are subgroups of G. (Different groups and subgroups from #2!) Suppose further that H is *not* a subgroup of K and K is *not* a subgroup of H. Choose $a \in H, a \notin K$ and $b \in K, b \notin H$.

Prove that $ab \notin H \cup K$. (This explains why $H \cup K$ is not a subgroup of G.) Hint: if $ab \in H \cup K$, then $ab \in H$ or $ab \in K$. Try to find a contradiction.

4. ($\mathcal{E}$) Suppose that G_1 and G_2 are isomorphic groups and $\Phi : G_1 \to G_2$ is an isomorphism. Suppose H is a subgroup of G_1 and let

$$\Phi(H) := \{\Phi(h) : h \in H\} \subseteq G_2.$$

denote the image of H under Φ. Prove carefully that $\Phi(H)$ is a subgroup of G_2. You already know (and do **not** have to prove) that $\Phi(H)$ is a subset of G_2, but there are other properties that you have to prove.

5. *Fraleigh* §1.8 (p.84) – 20.

6. ($\mathcal{E}$) You are given the following two permutations

$$\sigma = \begin{pmatrix} 12345 \\ 34152 \end{pmatrix} = (13)(245), \qquad \tau = \begin{pmatrix} 12345 \\ 13452 \end{pmatrix} = (2345).$$

Compute $\sigma\tau$ and $\tau\sigma$ and determine the smallest integer k so that σ^k is the identity.