Math 417 **Homework 3** **Due Friday, February 8, 2019**

It's ok to work together, but don't copy without comprehension. The symbol $(\mathcal{E})$ means that at least part of this problem is a former exam question.

Ungraded. The answers to these are in the back, but I will write full solutions and you are "responsible" for them: *Fraleigh* §1.5 (p.56) – 21; §1.6 (p.66) – 17, 27.

1a. *Fraleigh* §1.4 (p.48) 29. Hints: For any group G and any $x \in G$, $(x^{-1})^{-1} = x$, so you could think of pairs (x, x^{-1}). When is $x = x^{-1}$?

1b. *Fraleigh* §1.5 (p.58) 47. Don't forget that the set H is a subset of G, and G is a group, so various properties of G can be applied in H.

2. Determine all the subgroups of groups $((\mathbb{Z}/14\mathbb{Z})^*, \times)$ by listing the elements.

3. Give a careful and specific isomorphism between the group $((\mathbb{Z}/12\mathbb{Z})^*, \times)$ and the Klein four group V given by Table 5.11.

4. $(\mathcal{E})$ Let $G = \langle a \rangle$ be a cyclic group of order 18 with generator x. Determine (with explanation) the orders of each of these subgroups:

a. $H_1 = \langle a^{12} \rangle$;

b. $H_2 = \langle a^5 \rangle$.

5. $(\mathcal{E})$ Suppose Φ is an automorphism between groups $(G, *_G)$ and $(H, *_H)$, where the subscripts indicated the operations of the respective groups.

a. $(\mathcal{E})$ Let e_G and e_H denote the identity elements of G and H. Prove carefully that $\Phi(e_G) = e_H$. (Yes, I did this in class.)

b. Prove that for $g \in G$, $\Phi(g^{-1}) = (\Phi(g))^{-1}$.

c. Suppose $g \in G$ has order k. Prove carefully that $\Phi(g) \in H$ also has order k.

Hint: this is not a hard problem if you know the definitions.

6. $(\mathcal{E})$ Consider the following two permutations in S_3:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \qquad \tau = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

Determine σ^2, $\sigma\tau$, $\tau\sigma$ and τ^2.