Math 417 **Homework 2** **Due Friday, February 1, 2019**

It's ok to work together, but don't copy without comprehension. The grader and I can often detect this! The symbol ($\mathcal{E}$) means that at least part of this problem is a former exam question. No ungraded problems this week.

Ungraded. The answers to these are in the back, so they won't be graded, but I will write full solutions and you are "responsible" for them: *Fraleigh* §1.2 (p.26) – 7, 9; §1.3 (p.34) – 3,5; §1.4 (p.45) – 3,5, 31.

1a. *Fraleigh* §1.3 (p.34) – 6.
1b. *Fraleigh* §1.4 (p.44) – 4. (If it fails, list all axioms it fails!)

2. ($\mathcal{E}$) Let $S = \mathbb{R}\setminus\{1\}$, that is, the set of all real numbers except 1. Define a binary operation of S by

$$a * b = a + b - ab,$$

where the operations on the right are the usual arithmetic ones, so that, for example $2 * 3 = 2 + 3 - 6 = -1$. Prove that $(S, *)$ a group. For this problem, you need to check that each part of the definition on pp.37-38 is valid. You will need to find the identity element e and the formula for the inverse element.

3a. ($\mathcal{E}$) Suppose $n = 168 = 2^3 \cdot 3 \cdot 7$. Determine $\nu_2(n)$, $\nu_3(n)$ and $\nu_5(n)$ (not a typo!)
3b. ($\mathcal{E}$) Suppose p is a prime and $p^3 \mid n^2$. Prove that $p^4 \mid n^2$.
3c. Find a *non-prime* a and an integer n so that $a^3 \mid n^2$, but a^4 does *not* divide n^2. (Hint: try (a, n) so that $a^3 = n^2$.)

4 and 5. ($\mathcal{E}$) Define in the usual way the set

$$S = (\mathbb{Z}/5\mathbb{Z})^* = \{[1]_5, [2]_5, [3]_5, [4]_5\},$$

and define the binary operation $*$ on S by $[a]_5 * [b]_5 = [3ab]_5$. (For example $[1]_5 * [4]_5 = [3 \cdot 1 \cdot 4]_5 = [12]_5 = [2]_5$.) For your convenience, the multiplication table is given on this sheet. (The table shows that $*$ is a binary operation, and you do **not** have to prove this!) Your task is to prove that $G = (S, *)$, with **this** strange definition, is a group.
a. (Write down a formula explaining what "associative" means and use it to explain why $*$ is associative in G.
b. Determine the identity element in G. (The table will be helpful.)
c. Find the inverses of the four elements in G. (I want four answers here; the table will be helpful again.)
d. Give a specific isomorphism between S and the usual additive group $T = (\mathbb{Z}/4\mathbb{Z})^*$. It is enough to determine the image in T of the four elements of S. There are two correct solutions; you do not have to find both!

6. Give a careful and specific isomorphism between the groups $((\mathbb{Z}/12\mathbb{Z})^*, \times)$ and the group V given by Table 5.11.

For #4,5

[a] * [b]

*	[1]	[2]	[3]	[4]
[1]	3	1	4	2
[2]	1	2	3	4
[3]	4	3	2	1
[4]	2	4	1	3