Math 417 **Homework 5** **Due Friday, October 2, 2020, 11:59pm**

It's ok to work together, but don't copy without comprehension. The symbol ($\mathcal{E}$) means that at least part of this problem is a former exam question.

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1. ($\mathcal{E}$) Suppose G is a group and H and K are subgroups of G. Prove that $H \cap K$ is a subgroup of G. (It is possible that $H \cap K = \{e\}$.)

2/3 (Counts as two problems) ($\mathcal{E}$) Let $G = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$; the operation in G is addition.) Let $H = \langle a \rangle, a^{12} = e$, be a cyclic group of order 12. Let ϕ be a homomorphism from G to H defined by

$$\phi([i]_2, [j]_4) = a^{6i+3j}; \qquad 0 \le i \le 1, \quad 0 \le j \le 3.$$

a. Compute ϕ for the eight elements of G.

b. Determine $K = Ker(\phi) \subseteq G$ and $Im(\phi) \subseteq H$.

c. Write down the cosets of K in G.

d. Give the multiplication table of the factor group G/K.

4. ($\mathcal{E}$) Suppose G is an abelian group and let $L = G \times G = \{(g_1, g_2) \mid g_1, g_2 \in G\}$, with the usual operation on L. Define a map $\phi : L \mapsto G$ by

$$\phi(g_1, g_2) = g_1^2 g_2.$$

a. Prove that ϕ is a homomorphism from L to G.
b. For $g \in G$, prove that there is a unique $h \in G$ so that $(g, h) \in Ker(\phi)$, and describe h in terms of g,

5. ($\mathcal{E}$) Suppose $K = \{e, a, b\}$ is a normal subgroup of a group G. Here, I mean that e, a and b are different elements. Suppose $g \in G$ has the property that $ga \ne ag$, Prove that $ga = bg$. (Hints: (i) Don't panic! (ii) What does it mean for K to be a normal subgroup?)

The following problems are *ungraded* , but are suggested review, especially for people who had difficulty with related problems on various homeworks. I will look at anything you choose to submit with the rest of the HW5 or you can also email them to me directly:

6. Write the following two permutations in S_9 in the cycle notation.

$$\pi_1 = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ 5\,6\,1\,9\,2\,3\,4\,8\,7 \end{pmatrix} \qquad \pi_2 = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ 3\,6\,5\,2\,9\,8\,1\,7\,4 \end{pmatrix}$$

Also, compute $\pi_1\pi_2$, making sure you choose the correct order.

7. Someone tells you, accurately, that $((\mathbb{Z}/19\mathbb{Z})^*, \odot)$ is a cyclic group of order 18, and that one generator is $[14]_{18}$. Express all the generators of this group in the form $[14^k]_{18}$. If you take any numerical powers of 14, then you are working much too hard!

8. Define the binary operator $*$ on the set of positive real numbers by $a * b = \sqrt{ab}$. Is $*$ associative?