Math 417 **Homework 4** **Due Friday, September 25, 2020, 11:59pm**

It's ok to work together, but don't copy without comprehension. The symbol $(\mathcal{E})$ means that at least part of this problem is a former exam question.

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1. $(\mathcal{E})$ Suppose G is an *abelian* group and let H denote the set of squares in G. That is,

$$H = \{g^2 : g \in G\} = \{g * g : g \in G\}.$$

Prove carefully that H is a subgroup of G. You will need to use the fact that multiplication in G is commutative! (Don't worry about deciding whether H is a proper subgroup; sometimes it is, sometimes it isn't, and don't worry about associativity, which is automatic.)

2. Let $G = \mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z}$, so the elements of G are $([i]_6, [j]_8)$, where $0 \leq i \leq 5,\ 0 \leq j \leq 7$. Let $H = \{([0]_6, [0]_8), ([0]_6, [4]_8), ([3]_6, [0]_8), ([3]_6, [4]_8)\}$ be a subgroup of G. (You aren't asked to prove that H is a subgroup of G.)

a. Determine the number of different left cosets of H in G.
b. Write down three *different* left cosets of H.

3. $(\mathcal{E})$ Let $G = D_4$ be the dihedral group with 8 elements. Recall that the elements of D_4 are:

$$\{\rho_0 = (1)(2)(3)(4), \rho_1 = (1234), \rho_2 = (13)(24), \rho_3 = (1432),$$
$$\mu_1 = (12)(34), \mu_2 = (14)(23), \delta_1 = (13)(2)(4), \delta_2 = (1)(24)(3)\}$$

Determine the left and right cosets of the subgroup $H_1 = \{\rho_1, \mu_1\}$.

4. $(\mathcal{E})$ Suppose G is a group whose elements are

$$\{a^j b^k \mid j \in \{0, 1, 2, 3, 4, 5, 6, 7\},\ k \in \{0, 1\}, \}$$

and satisfying the following rules

$$a^8 = 1, \quad , b^2 = 1, \quad ba = a^7 b$$

I am *not* asking for the full multiplication table. I want you to write

$$ba^2, \quad ba^3, \quad ba^4$$

in the form $a^j b^k$ for specific (j, k). Use your results to compute

$$(a^3 b)^2, \quad (a^6 b)(a^2 b).$$

5. $(\mathcal{E})$ Suppose G is a group and H an d K are subgroups of G. Suppose further that H is *not* a subgroup of K and K is *not* a subgroup of H. Accordingly, let $a \in H, a \notin K$ and $b \in K, b \notin H$.

Prove that $ab \notin H \cup K$. (This explains why $H \cup K$ is not a subgroup of G.) Hint: if $ab \in H \cup K$, then $ab \in H$ or $ab \in K$. Try to find a contradiction.