1. (E) Suppose $G$ is an abelian group and let $H$ denote the set of squares in $G$. That is, 
$$H = \{g^2 : g \in G\} = \{g \cdot g : g \in G\}.$$ 
Prove carefully that $H$ is a subgroup of $G$. You will need to use the fact that multiplication in $G$ is commutative! (Don’t worry about deciding whether $H$ is a proper subgroup; sometimes it is, sometimes it isn’t, and don’t worry about associativity, which is automatic.)

2. Let $G = \mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z}$, so the elements of $G$ are $(i, j)$, where $0 \leq i \leq 5$, $0 \leq j \leq 7$. Let $H = \{(0,0),(0,4),(3,0),(3,4)\}$ be a subgroup of $G$. (You aren’t asked to prove that $H$ is a subgroup of $G$.)

a. Determine the number of different left cosets of $H$ in $G$.

b. Write down three different left cosets of $H$.

3. (E) Let $G = D_4$ be the dihedral group with 8 elements. Recall that the elements of $D_4$ are:
$$\rho_0 = (1)(2)(3)(4), \rho_1 = (1234), \rho_2 = (13)(24), \rho_3 = (1432),$$
$$\mu_1 = (12)(34), \mu_2 = (14)(23), \delta_1 = (13)(2)(4), \delta_2 = (1)(24)(3)$$
Determine the left and right cosets of the subgroup $H_1 = \{\rho_1, \mu_1\}$.

4. (E) Suppose $G$ is a group whose elements are
$$\{a^j b^k \mid j \in \{0, 1, 2, 3, 4, 5, 6, 7\}, k \in \{0, 1\}, \}$$
and satisfying the following rules
$$a^8 = 1, \ b^2 = 1, \ ba = a^7b$$
I am not asking for the full multiplication table. I want you to write
$$ba^2, \ ba^3, \ ba^4$$
in the form $a^j b^k$ for specific $(j, k)$. Use your results to compute
$$(a^3b)^2, \ (a^6b)(a^2b).$$

5. (E) Suppose $G$ is a group and $H$ and $K$ are subgroups of $G$. Suppose further that $H$ is not a subgroup of $K$ and $K$ is not a subgroup of $H$. Accordingly, let $a \in H, a \notin K$ and $b \in K, b \notin H$.

Prove that $ab \notin H \cup K$. (This explains why $H \cup K$ is not a subgroup of $G$.) Hint: if $ab \in H \cup K$, then $ab \in H$ or $ab \in K$. Try to find a contradiction.