Math 417 **Homework 3** **Due Friday, September 18, 2020, 11:59pm**

It's ok to work together, but don't copy without comprehension. The symbol $(\mathcal{E})$ means that at least part of this problem is a former exam question.

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1. $(\mathcal{E})$ Don't answer this question by citing the multiplication table for S_3, although I can't stop you from checking your answer. Let

$$\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}.$$

Determine α^2, $\alpha\beta$, $\beta\alpha$, and β^2.

2. We have already seen that $G = ((\mathbb{Z}/9\mathbb{Z})^*, \odot)$ and $H = ((\mathbb{Z}/14\mathbb{Z})^*, \odot)$ are each cyclic groups of order 6. Write out, with explanation, an explicit isomorphism Φ from G to H. It should include six equations that look like $\Phi([a]_9) = [b]_{14}$, with specific integers for a, b.

3. $(\mathcal{E})$ You are given the following two permutations in S_5

$$\sigma = \begin{pmatrix} 12345 \\ 13425 \end{pmatrix} = (1)(234)(5), \qquad \tau = \begin{pmatrix} 12345 \\ 34152 \end{pmatrix} = (13)(245).$$

Compute $\sigma\tau$ and $\tau\sigma$ and determine the smallest positive integer k so that τ^k is the identity.

4. $(\mathcal{E})$ It is an arithmetic fact that

$$10^6 - 1 = 999999 = 999 \cdot 1001 = (3^3 \cdot 37) \cdot (7 \cdot 11 \cdot 13) \quad \text{and} \quad 5291 = 11 \cdot 13 \cdot 37.$$

Using this information, and without doing any more calculation, determine the decimal expansion of

$$\frac{400}{5291}.$$

5. $(\mathcal{E})$ Someone tells you, correctly, that $((\mathbb{Z}/13\mathbb{Z})^*, \odot)$ is a cyclic group of order 12 and $[2]_{13}$ is a generator; that is, $\langle [2]_{13} \rangle = (\mathbb{Z}/13\mathbb{Z})^*$.

a. Determine all generators of $((\mathbb{Z}/13\mathbb{Z})^*, \odot)$.

b. Since 3 and 4 are divisors of 12, a cyclic group order 12 has a subgroup of order 3 and a subgroup of order 4. Write down the elements of the subgroups of $((\mathbb{Z}/13\mathbb{Z})^*, \odot)$ of order 3 and of order 4.

You will find it helpful to recall that for a cyclic group $C_n = \langle a \rangle$, we already know the formula $\langle a^k \rangle = \frac{n}{gcd(k,n)}$.