Math 417 **Homework 1** **Due Friday, September 4, 2020**

It's ok to work together, but don't copy without comprehension. It's also ok to type up your answers, as long as you put your student number on it. The symbol ($\mathcal{E}$) means that at least part of this problem is a former exam question. More information about submitting problems will be available early next week.

1a. ($\mathcal{E}$) Compute $g = \gcd(12, 20)$ by the Euclidean algorithm and find integers r, s so that $g = 12r + 20s$.

1b. Let $G = C_{20} = \langle a \rangle$ be a cyclic group of order 20. Using your answer from 1a., or any correct method, compute the number of elements in $H = \langle a^{12} \rangle$, the subgroup of powers of a^{12} in G.

1c. List the elements in any **proper** subgroup of H (that is, give me *one* subgroup of H that is *not* $\{e\}$ or H.)

2a. Determine $(\mathbb{Z}/14\mathbb{Z})^*$. (Hints: $14 = 2 \cdot 7$ and this set has 6 elements. It is ok in this problem to write your solution with, say "1" standing in for $[1]_{14}$.)
2b. Write down the multiplication table for $(\mathbb{Z}/14\mathbb{Z})^*$.

2c. Which (if any) elements a have the property that the order of a is 6? That is, $a^6 \equiv 1 \mod 14$, but $a^j \not\equiv 1 \mod 14$ for $j \in \{1, 2, 3, 4, 5\}$.

3 ($\mathcal{E}$). The following is a bad (but accurate) multiplication table of a group G. You may assume this is a group, **you are not asked to prove this!** All questions can be answered by reading the table.

*	u	v	w	x
u	v	x	u	w
v	x	w	v	u
w	u	v	w	x
x	w	u	x	v

3a. Which element is the identity in the group? (Hint: it isn't u, even though it's the first element of the table, because this is a bad table.)

3b. Determine the inverses of u, v, w, x.

3c. Find a specific isomorphism Φ from G to your favorite C_4.

4. ($\mathcal{E}$) Find two sets of integers (a_i, b_i, c_i), $i = 1, 2$ so that $\gcd(a_i, b_i) = 4$ and $\gcd(a_i, c_i) = 6$, but $\gcd(b_1, c_1) \neq \gcd(b_2, c_2)$.

5. Use your imagination and invent a situation in which one of the objects is a cyclic group of order 7. (But not simply adding integers mod 7 or rotating a regular 7-gon.) I have no specific answer in mind. Surprise me!