

Worksheet

1. Determine the order of the element in \mathbb{Q}_8

a) Find subgroups.

b) Is any subgroup isomorphic to V ?

2. In each case, either find a, b, c with the properties or explain why it's impossible.

i) $\gcd(a, b) = 1$
 $\gcd(a, c) = 2$
 $\gcd(b, c) = 4$

ii) $\gcd(a, b) = 1$
 $\gcd(a, c) = 2$
 $\gcd(b, c) = 3$

see other page for details on \mathbb{Q}_8 .

	$1 - 1$	$i - i$	$j - j$	$k - k$
1	$1 - 1$	$i - i$	$j - j$	$k - k$
-1	$-1 1$	$-i i$	$-j j$	$-k k$
i	$i - i$	$-1 1$	$k - k$	$-j j$
$-i$	$-i i$	$1 - 1$	$-k k$	$j - j$
j	$j - j$	$-k k$	$-1 1$	$i - i$
$-j$	$-j j$	$k - k$	$1 - 1$	$-i i$
k	$k - k$	$j - j$	$-i i$	$-1 1$
$-k$	$-k k$	$-j j$	$i - i$	$1 - 1$

$1 = \text{identity}$, -1 commutes with everything

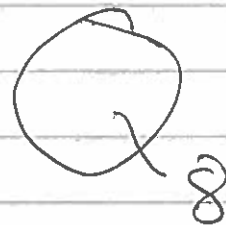
$$i^2 = j^2 = k^2 = -1$$

The Quaternion Group
of Hamilton.

$$ij = k \quad ji = -k$$

$$jk = i \quad kj = -i$$

$$ki = j \quad ik = -j$$



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