Math 417
1/18/19

The Euclidean Algorithm in action!

$n_0 = 417 \quad n_1 = 345$

$$417 = 1 \cdot 345 + 72 \quad (\text{so } n_2 = 72)$$
$$345 = 4 \cdot 72 + 57 \quad (\text{so } n_3 = 57)$$
$$72 = 1 \cdot 57 + 15 \quad n_4 = 15$$
$$57 = 3 \cdot 15 + 12 \quad n_5 = 12$$
$$15 = 1 \cdot 12 + 3 \quad n_6 = 3$$
$$12 = 4 \cdot 3 + 0 \longleftarrow \text{so we stop}$$

$$\gcd(345, 417) = 3$$

Also: $3 = 15 - 12 = 15 - (57 - 3 \cdot 15) = 4 \cdot 15 - 1 \cdot 57$

(we go back up the ladder)

$$3 = 4 \cdot 15 - 1 \cdot 57 = 4 \cdot (72 - 57) - 57 = 4 \cdot 72 - 5 \cdot 57$$
$$= 4 \cdot 72 - 5(345 - 4 \cdot 72) = (4 + 4 \cdot 5)\, 72 - 5 \cdot 345$$
$$= (24)(72) - 5 \cdot 345$$
$$= 24(417 - 345) - 5 \cdot 345$$
$$= 24 \cdot 417 - 29 \cdot 345 \quad (10008 - 10005!)$$

Suppose $g \mid 417$ and $g \mid 345$, Then $g \mid 72$, $g \mid 57$, $g \mid 15$, $g \mid 12$ and $g \mid 3$ by looking at the equations and walking down the ladder.
On the other hand $3 \mid 12$ (That's why the algorithm stopped.)
So $3 \mid 15$, $3 \mid 57$, $3 \mid 72$, $3 \mid 345$, $3 \mid 417$ by walking up the latter. We'll prove this is true in general