1. Express \( f(z) = \frac{1}{z^2 + 3} \) as a Taylor series at \( z = 1 \).

2. (E) Let

\[
f(z) = \frac{1}{1 - 2z} + \frac{1}{2 + z}.
\]

Express \( f \) as a Laurent series in \( z + 2 \) which converges for \(|z + 2| > \frac{5}{2}\).

3. Express the \( f \) from the last problem as a Laurent series in \( z + 2 \) which converges for \( 0 < |z + 2| < \frac{5}{2} \).

4. §3.8 – 1 (first five) – discuss the singularities at \( \infty \).

5. (E) Classify all singularities (including at \( \infty \)) of the function

\[
f(z) = \frac{1}{e^{z^2} - 1}.
\]

6. (E) Classify all singularities (including at \( \infty \)) of the function

\[
f(z) = \frac{(z - 2)^2 \sin \frac{1}{z}}{z^3 - 4z}.
\]

7. Name one of the authors of our textbook.

8. (E) Suppose \( f \) and \( g \) are entire functions and neither is identically zero. Suppose further that, for all \( z \), \(|f(z)| \leq 2|g(z)|\).

a. Show that the only singularities of \( h = \frac{f}{g} \) are removable ones at the zeros of \( g \). (Hint: you know something about \( h \) in a neighborhood of a zero of \( g \).)

b. Prove that there is a constant \( c \) so that \( f(z) = cg(z) \) for all \( z \). (Hint: use (a) and an important theorem from class, applied to an entire function that is usually equal to \( h \).)

9. §3.8 – 6 (first two).

10. Suppose \( p(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_0 \) is a polynomial. Find \( R = R(a_{n-1}, \ldots, a_0) \) so that \(|z| \geq R\) implies that \(|p(z)| \geq .99|z|^n\). (See handout of 2/25/00.)

11. Name the other author of the textbook. (It pays to read the last four problems!)

12. p.174 – 7.2. (Hint: Example 2.1, p. 13.)