(1) Let \( \{ A_i \}_{i \geq 0} \) be abelian groups. Define functors \( F, G : \mathbb{N}^{op} \to \text{Ab} \) as follows.

\[
F(k) := \bigoplus_{i \geq k} A_i, \quad f : F(k + 1) \to F(k) \text{ is inclusion}, \\
G(k) := \prod_{i \geq k} A_i, \quad g : G(k + 1) \to G(k) \text{ is inclusion}.
\]

Let \( H = \text{Cok}(\alpha) \), where \( \alpha : F \to G \) is the evident monomorphism of functors.

(a) Show that \( \lim H \approx \lim^1 F \).

(b) Show that \( \lim H \approx H(0) \).

(c) Exhibit a collection of abelian groups \( \{ A_i \} \) together with a non-zero element of \( \lim^1 F \).

(2) Let \( M \in R\text{-mod} \). Recall that \( - \otimes_R M : \text{mod}-R \to \text{Ab} \) preserves arbitrary direct sums, and therefore preserves finite products.

(a) Show that if \( M \) is finitely presented, then \( - \otimes_R M \) preserves arbitrary products.

(b) Show the converse: if \( M \in R\text{-mod} \) is such that \( - \otimes_R M \) preserves arbitrary products, then \( M \) is finitely presented. (Hint: to show \( M \) finitely generated, consider tensor product with \( \prod_{\alpha \in S} R \) for a suitably chosen indexing set \( S \); to show \( M \) finitely presented, use an exact sequence \( 0 \to N \to R^p \to M \to 0 \) and show \( N \) must be finitely generated.)

(3) Let \( k \) be a field, and let \( R = k[x]/(x^2) \). Regard \( k \) as an \( R \)-module via \( k \approx R/(x) \).

Compute \( \text{Tor}_q^R(k, k) \) and \( \text{Ext}_q^R(k, k) \) for all \( q \geq 0 \), using projective resolutions.

(4) Let \( k \) be a field, and let \( R = k[x]/(x^3) \). Regard \( k \) as an \( R \)-module via \( k \approx R/(x) \). Let \( M = R/(x^2) \).

Compute \( \text{Tor}_q^R(k, k), \text{Tor}_q^R(M, k), \text{Ext}_q^R(k, k), \text{Ext}_q^R(M, k) \) using projective resolutions.

(5) Give an explicit example of (i) a complex \( P \in \text{Ch}_{\geq 0}(\text{Ab}) \) of projectives, (ii) a projective resolution \( Q \to H_0 P \) of the 0th homology of \( P \), and (iii) two chain maps \( f, g : Q \to P \) which induce isomorphisms in \( H_0 \), but which are not chain homotopic.