(1) Describe a preadditive category $C$ with the property that $\text{Fun}^{\text{add}}(C,A) \approx \text{Ch}(A)$ for any preadditive category $A$.

(2) Let $\mathcal{R}$ be a pre-additive category with a finite set of objects $\{X_1, \ldots, X_n\}$. Let $R := \prod_{i,j=1}^n \text{Hom}_\mathcal{R}(X_j, X_i)$, i.e., $R$ is the set of $n^2$-tuples $(r_{ij})_{i,j=1}^n$ where $r_{ij} : X_j \to X_i$ in $\mathcal{R}$.

(a) Define a product on $R$, show that your product makes $R$ into a an associative ring, and prove that there is an equivalence of categories $R\text{-mod} \approx \text{Fun}^{\text{add}}(\mathcal{R}, \text{Ab})$.

(b) Let $A$ be a ring, and suppose $\mathcal{R}$ is defined by $\text{Hom}_\mathcal{R}(X_j, X_i) = A$ for all $1 \leq i, j \leq n$, with composition defined by multiplication in $A$. Show that if $n \geq 1$, there is an equivalence of categories $\text{Fun}^{\text{add}}(\mathcal{R}, \text{Ab}) \approx A\text{-mod}$, defined on objects by $F \mapsto F(X_1)$ and on maps by $f \mapsto f(X_1)$.

(c) Show that $A\text{-mod} \approx M_{n \times n}(A)\text{-mod}$ for any ring $A$ and $n \geq 1$.

(3) Show that the category of finitely generated free abelian groups is an additive category such that every map has a kernel and a cokernel, but that it is not an abelian category. (Hint: cokernels are not necessarily quotient groups.)

(4) Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be abelian categories. Show that $\mathcal{C} := \mathcal{A}_1 \times \mathcal{A}_2$ is an abelian category, and that the projection and inclusion functors $\mathcal{A}_1 \rightleftharpoons \mathcal{C} \rightleftharpoons \mathcal{A}_2$ are exact.

(5) Let $R_1, R_2$ be rings, and let $R = R_1 \times R_2$ be the product ring. Show that there is an equivalence of categories $R\text{-mod} \approx R_1\text{-mod} \times R_2\text{-mod}$.