Mathematics 428 — Honors Topics in Mathematics
Introduction to Galois Theory

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About the course: This course provides an introduction to Galois Theory. This theory was developed to answer questions of the following type: “How can find the solution of a polynomial equation \( f(x) = 0 \) in one variable? Can you solve it using a specific list of operations (such as: the operations of arithmetic together with taking \( n \)th roots)?”

Prerequisites: Ability to write and follow mathematical proofs. Some knowledge of abstract algebra, such as the notions of group, field, and ring, as presented in Math 417 or Math 427. The basic notions needed will be reviewed.


Topics: The main goal is to prove some of the classic insolubility results for polynomial equations, notably the insolubility of the 5th degree equation in terms of radicals. We will also prove the impossibility of solving certain geometric problems by ruler and compass (such as trisecting an angle, “duplicating the cube”, constructing the regular 7-gon, and so forth).

Proving these will involve developing the theory of field extensions, leading to a proof of the “Galois correspondence” relating field extensions of \( F \) to subgroups of the Galois group of \( F \).

If time permits, we’ll discuss other topics, including finite fields and applications of Galois theory to number theory.

Format:
The class will be lecture and discussion based.

There will be:

- **problem sets**, usually due every other week. A subset of the problems in each set will be graded.

- a **take-home midterm exam**, and

- a **take-home final exam**.

I reserve the option to replace either the midterm or final exam with some other kind of project.

Grades:
Problem sets: 50%, Midterm 20%, Final 30%. Grades will be posted on the course page at https://learn.illinois.edu.