Due in class F 5 Oct.
Note: clarified statement of (4) [W 3 Oct].

(1) Let $\mathbb{Q}$ be the rationals, as a group under addition.
   (a) Show that any finitely generated subgroup of $\mathbb{Q}$ is contained in a cyclic subgroup.
       (Hint: Write a sum of fractions in terms of a common denominator. Think about
        a special case like $\langle \frac{1}{2}, \frac{2}{3} \rangle$.)
   (b) Show that $\mathbb{Q}$ is not a cyclic group.
   (c) Use the above to show that $\mathbb{Q}$ is not finitely generated.

(2) Give a formula for the number of $k$-cycles in $S_n$ ($1 \leq k \leq n$), and show why your
    formula is correct.

(3) Describe all possible cycle types in $S_5$, and count the number of permutations of each
    type.

(4) Let $\sigma \in S_n$. Show that if $\sigma^d = id$ for some $d \geq 1$, then the decomposition of $\sigma$
    into disjoint cycles can only involve cycles whose order divides $d$.

(5) Let $X$ be a finite set of size $n = |X|$. Let $\sigma \in \text{Sym}(X)$ be a permutation of $X$ with
    the property that $\sigma^p = id$ for some prime number $p$. Define
    $$X^\sigma := \{ x \in X \mid \sigma(x) = x \}.$$ 
    Show that
    $$|X^\sigma| \equiv |X| \mod p.$$