Lagrange Multipliers and Constrained Optimization (Merit)

Thursday, February 22nd

1. Use the method of Lagrange to find all points on the hyperbola $xy = 1$ that lie closest to the origin.

2. Find the maximum value of $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the surfaces $x - y + z = 1$ and $x^2 + y^2 = 1$.

3. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraints:

   (a) $f(x, y, z) = x^2y^2z^2$ subject to $x^2 + y^2 + z^2 = 1$.

   (b) $f(x_1, x_2, \ldots, x_n) = x_1 + x_2 + \cdots + x_n$ subject to $x_1^2 + x_2^2 + \cdots + x_n^2 = 1$.

4. Find the extreme values of $f(x, y) = e^{-xy}$ on the region described by the inequality $x^2 + 4y^2 \leq 1$.

5. Find the maximum and minimum value of the function $f(x, y) = 2x + 3y + 4$ on the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

6. (Optional) In figure 1 light travels through medium 1 at speed v and medium 2 at speed w. Assuming that light takes the fastest possible path from A to B, how do α and β depend on v and w? Optional hint: look up (or ask me about) Snell's law.

7. (Optional) Use Lagrange multipliers to prove that the triangle of fixed perimeter $2s$ which attains maximum area is actually an equilateral. (*Hint*: Use Heron's formula for the area: $A = \sqrt{s(s-x)(s-y)(s-z)}$, where x, y, z are the lengths of the sides.)

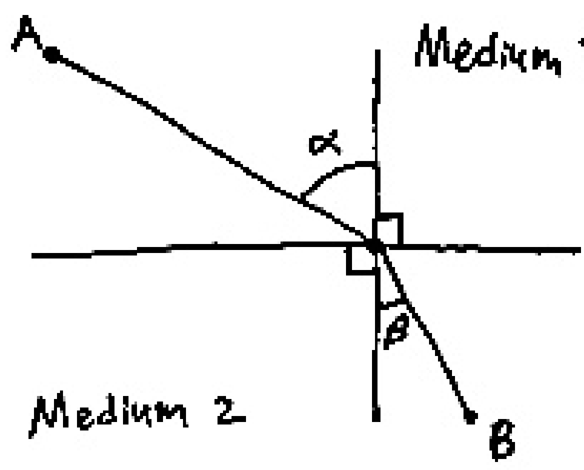

Figure 1: The speed of light in Medium 1 is v and the speed of light in Medium 2 is w.