Chapter 1  Real Numbers

Section 1.1  Sets of Real Numbers

The natural numbers are sometimes called the _counting_ numbers; they are:

\[ 1, 2, 3, 4, 5, 6, \ldots \]

The whole numbers are:

\[ 0, 1, 2, 3, 4, 5, 6, \ldots \]

The integers are the whole numbers and their _opposites_; they are:

\[ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \]

Rational numbers are any numbers that can be written as the _ratio_ of two integers. (the second being nonzero)

Note: The decimal form of a rational number is a _terminating_ or _repeating_ decimal.

Irrational numbers are real numbers that are _not_ rational!!

Examples of Rational Numbers:

\[ 0, \quad \frac{1}{2}, \quad 1, 0.125, \quad 1.045, \quad -\frac{8}{3}, \quad -0.\overline{9}, \quad -\sqrt{4}, \quad 3\sqrt{-8} \]

Examples of Irrational Numbers:

\[ \sqrt{2}, \quad \sqrt{6}, \quad \pi, \quad e, \quad -\sqrt{5} \]

Note: All integers are also rational numbers. \[ 0 = \frac{0}{1} \]

Note: Every real number is either _rational_ or _irrational_.

Are there any numbers that are not real numbers?? \[ \sqrt{-1}, \quad \ell, \quad \phi, \quad \infty, \quad -\sqrt{-2} \]
**Example** Consider the set $S = \{6, \sqrt{36}, \sqrt[3]{36}, -\sqrt{36}, \sqrt{-36}, -\frac{5}{6}, \frac{0}{6}, \frac{6}{0}, 0.\overline{6}, 0.676767677\ldots\}$

List all elements of $S$ which are

a) Natural numbers $6$ $\sqrt{36}$

b) Whole numbers $6$ $\sqrt{36}$

c) Integers $6$ $\sqrt{36}$ $-\sqrt{36}$

d) Rational numbers $6$ $\sqrt{36}$ $-\sqrt{36}$ $-\frac{5}{6}$ $\frac{0}{6}$ $0.\overline{6}$

e) Irrational numbers $\sqrt{6}$ $0.676767677\ldots$

f) Real numbers $6$ $\sqrt{6}$ $\sqrt{36}$ $-\sqrt{36}$ $-\frac{5}{6}$ $\frac{0}{6}$ $0.\overline{6}$ $0.676767677\ldots$

**Properties of Real Numbers**

The **commutative** properties have to do with ____________.

- Commutative Property of Addition: $a + b = b + a$
- Commutative Property of Multiplication: $a \cdot b = b \cdot a$

The **associative** properties have to do with ____________.

- Associative Property of Addition: $(a + b) + c = a + (b + c)$
- Associative Property of Multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

The **Distributive Property of Multiplication over Addition**:

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$
Additive Identity Property:

\[ a + 0 = a \]

Note: 0 is called the additive identity.

Multiplicative Identity Property:

\[ a \cdot 1 = a \]

Note: 1 is called the multiplicative identity.

Additive Inverse Property:

\[ a + (-a) = 0 \]

Note: The additive inverse of \( a \) is \(-a\).

Multiplicative Inverse Property:

\[ a \cdot \frac{1}{a} = 1 \quad \text{if} \; a \neq 0 \]

Note: The multiplicative inverse of \( a \) is \( \frac{1}{a} \).

Example: The multiplicative inverse of \( \frac{2}{3} \) is \( \frac{3}{2} \).

Identify the property illustrated:

a) \( 2 + (y + 0) = (2 + y) + 0 \) \( \text{Assoc. +} \)
b) \( 2 + (y + 0) = 2 + y \) \( \text{+ Identity} \)
c) \( 2 + (y + 0) = 2 + (0 + y) \) \( \text{+ comm.} \)
d) \( 5(2x - 3) = 10x - 15 \) \( \text{distributivity} \)
e) \( \frac{x+y}{3} \cdot \frac{3}{x+y} = 1 \) \( \text{mult. inverses} \)
f) \( (x-1)(x+1) = (x+1)(x-1) \) \( \text{mult. comm.} \)
g) \( 2a + 3a = (2 + 3)a \) \( \text{dist.} \)
Section 1.1  Exercises

For exercises #1 - #6 consider this set of numbers: \( S = \left\{ -\frac{3}{5}, -3, \frac{6}{3}, -\sqrt{3}, \sqrt{-3}, 3.6, 3.4, 0, \sqrt{8}, \frac{\pi}{6} \right\} \)

Which elements of \( S \) are . . .

1. . . . natural numbers?
2. . . . whole numbers?
3. . . . integers?
4. . . . rational numbers?
5. . . . irrational numbers?
6. . . . real numbers?

7. State three numbers that are integers but are not natural numbers.
8. State three numbers that are real numbers but are not rational numbers.
9. State three numbers that are rational numbers but are not integers.
10. State three numbers that are both integers and rational numbers.
11. State three numbers that are both whole numbers and integers.

\(#12 - #16\) State the additive inverse of each number:

12. 5
13. \(-m\)
14. \(\frac{1}{8}\)
15. \(\sqrt{7}\)
16. \(a - b\)

\(#17 - #21\) State the multiplicative inverse of each number:

17. \(-2\)
18. \(\frac{2}{3}\)
19. \(\frac{1}{4}\)
20. \(k\)
21. \(x + y\)

\(#22 - #27\) Name the property illustrated by each of the following:

22. \(3 \cdot 5 + 3 \cdot 9 = 3(5 + 9)\)
23. \(2(x + 3) = (x + 3) \cdot 2\)
24. \(1(x + 3) = x + 3\)
25. \(1(x + 3) = 1(3 + x)\)
26. \(2(3x) = (2 \cdot 3)x\)
27. \(\frac{5}{3} + \frac{5}{2} = 0\)
Section 1.1 Answers to Odd-Numbered Exercises

1. \( \frac{6}{3} \)

3. \( -3, \frac{6}{3}, 0 \)

5. \( -\sqrt{3}, \frac{\pi}{6} \)

7. Answers will vary.

9. Answers will vary.

11. Answers will vary.

13. \( m \)

15. \( -\sqrt{7} \)

17. \( \frac{1}{2} \)

19. \( -4 \)

21. \( \frac{1}{x+y} \)

23. Commutative Property of Multiplication

25. Commutative Property of Addition

27. Additive Inverse Property