Worksheet #4
Math 221 Lecture EL1

Instructions. Put your first and last name at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning. All worksheets from each group will be collected.

1. (a) List the ways in which the function $g(x) = \sqrt[3]{x} + x - 1$, the interval $[0, 1]$, and the number $N = 0$ satisfies the hypotheses of the IVT: the hypotheses are the requirements in the IF part of your statement from Problem 2, Worksheet 3, or the all the words between "Suppose" and "then" in the statement of the IVT in Section 2.5 on page 125 of your textbook.

Answer: The function $g(x) = \sqrt[3]{x} + x - 1$ is continuous on $[0, 1]$ because (i) root functions such as $\sqrt[3]{x}$ are continuous everywhere on their domains, as is the polynomial $x^2$ and the constant function $-1$. (and since $g(x)$ is a sum of continuous functions it will be continuous on its domain) (ii) the domain of $g(x)$ includes $[0, 1]$ because it is easy to see that we don’t break any algebra rules by plugging in numbers in $[0, 1]$. Now $g(0) = -1$ and $g(1) = 1$, so $g(0) \neq g(1)$, and $-1 < 0 < 1$, so this covers the requirements about $N$, and the requirements about $g(x)$ on $[0, 1]$.

(b) True or False: Does this mean that $g(x)$ has a root in the interval $(0, 1)$? Write the whole word True or False to receive credit. (Hint: use the Then part of the IVT).

Answer: True

2. Where is is the function

$$f(x) = \begin{cases} 
\sin(x) & \text{if } x < \frac{\pi}{4} \\
\cos(x) & \text{if } x > \frac{\pi}{4} \\
\frac{1}{\sqrt{2}} & \text{if } x = \frac{\pi}{4}
\end{cases}$$

continuous?

Answer: $(-\infty, \infty)$ (All real numbers)

3. Where is the function $\frac{1}{x+5}$ continuous?

Answer: $(-\infty, -5) \cup (-5, \infty)$, or all real numbers except $x = -5$. 

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