STUDY GUIDE FOR EXAM 2, LECTURE EL1

I have divided the study guide into Knowledge and Skills. Since I can only ask you to do a small number of problems in 50 minutes, this is a study guide, not a comprehensive list of all things you will perform on the exam. Also, there may be things not in this guide that you must know to answer exam questions because (1) they are considered prerequisite knowledge or skills or (2) the TAs and I saw no evidence that any students had not mastered them. Please consult me in office hours or your TAs in the tutor center if you feel unprepared on any of these points.

1. Knowledge for Exam 1

(1) Know what a theorem is. Know what type of logical statement it is (e.g. If-Then). Know the difference between the hypotheses and conclusion of a theorem.

(2) Know the Squeeze Theorem well enough to state it.

(3) Understand the relationship between the functions for the position \( p(t) \), velocity \( v(t) = p'(t) \), and acceleration \( a(t) = p''(t) \) in terms of first and second derivatives well enough to identify word problems that can be solved using differentiation.

(4) Know how to identify problems that you can use the Intermediate Value Theorem (IVT) to solve.

(5) Know which classes of functions are continuous on their domains; this requires a basic understanding of domains of these classes (See Theorem 7, page 123).

(6) Know which operations on continuous functions preserve continuity at a given point (Theorem 4, page 121).

(7) Know the limit definition of a derivative.

(8) Know Theorem 4 on page 158.

(9) Know how a function can fail to be differentiable (see page 159).

(10) Know from looking at a function \( f(x) \), which rule or combination of rules is the best choice to compute its derivative (Derivatives of Constants, The General Power Rule, the Constant Multiple Rule, the Sum and Difference Rules, and the Derivative of an Exponential Function, the Product Rule, and the Quotient Rule).

(11) If more than one rule applies to finding \( f'(x) \), know which is easiest.

(12) Know basic factoring formulas: difference of squares, sum of cubes, difference of cubes.

(13) Know both versions of the limit definition of a derivative.

(14) Know the definition of continuity. (Definition 1, Section 2.5, page 118).

(15) Know that if \( f(x) \) has a finite limit as \( x \to \pm\infty \), \( f(x) \) has at least one horizontal asymptote.

(16) Know how to identify vertical asymptotes of functions, if they exist.

(17) Know when rational functions do not have vertical asymptotes.

(18) Know the basic fact in Theorem 5 p. 133, as well as the facts

\[
\lim_{x \to \infty} \frac{1}{e^x} = 0,
\]

\[
\lim_{x \to -\infty} e^x = 0
\]

that motivate many of the algebraic manipulations we use to compute limits at positive and negative infinity.

(19) Know the common reasons that a function \( f(x) \) may not be differentiable at a certain point \( x = a \). (See page 159 of your book).
(20) Understand the relationship between the slope of a tangent line to a curve $f(x)$ at a point $x = a$ and $f'(a)$.

2. Knowledge for Exam 2

(1) Know the (pre-calculus level) laws of logarithms well, and the principles of trigonometric inverses (you can find all this in Section 1.6).
(2) Know how the laws of logarithms relate to logarithmic differentiation.
(3) Understand that some of the rates in related rates problems end up being variables you solve for to answer the word problem.
(4) Know ALL the properties of the exponential and logarithmic functions (Asympotes, their relationship, derivatives, ...).
(5) Understand why $\frac{d}{dx}a^x = a^x \ln a$ using the Chain Rule (see page 203).
(6) Understand the principle of implicit differentiation, when it is necessary, and why its use is the key to related rates problems.
(7) Knowing how to tell which quantities in a related rates problem are functions of $t$, and which are fixed constants.
(8) Understand how the signs of the first and second derivative relate to the shape of a graph.
(9) Know the different between local and global optima, and how to compute both kinds.
(10) Understand why different algebraic strategies help you compute limits of different types of indeterminate forms using L’Hospital’s rule.
(11) Know how to use basic geometry to relate quantities in word problems (similar triangles, areas, surface areas, and volumes of common solids and shapes, basic facts about triangles.
(12) Know the “trig circle of life” and definitions of all trigonometric functions in terms of sines and cosines.
(13) Know the fundamental trigonometric identity $\sin^2 x + \cos^2 x = 1$, and how it relates to the unit circle.
(14) Understand how Fermat’s Theorem relates to the critical numbers of a function.
(15) Know the precise definition of the critical numbers of a function $f$ (page 277).
(16) Understand how to effectively use critical numbers when solving problems (which ones are irrelevant due to a modeling problem, etc.)
(17) Know when and how to apply the First and Second Derivative Tests to obtain local maxima and minima.
(18) Know the definition of an inflection point (page 294).
(19) Know the correct place in the sequence of steps for solving a related rates problem (e.g.: usually after doing the implicit differentiation) and how this relates to the basic principles of treating derivatives as unknowns and implicitly defined equations.
(20) Know what concave up (“cup”) and concave down (“frown”) sections of a graph look like for increasing and decreasing functions.
(21) Know that $f'(c) = 0$ if $c$ is a local optima of $f$ and $f'(x)$ exists at $c$. (Page 276). Know what this looks like on a graph.
(22) Understand how Newton’s Law of Cooling can be modeled using the exponential growth/decay tools of 3.8.
(23) Understand the main ideas behind exponential growth and decay, especially Equation 1 and Theorem 2 on page 237.
3. Skills for Exam 1

(1) Compute the limits \(\lim_{x \to a^-} f(x)\), \(\lim_{x \to a^+} f(x)\), and \(\lim_{x \to a} f(x)\), either when they are equal to \(\pm \infty\), a finite number, or don’t exist for another reason, using a variety of algebraic techniques.

(2) Identify a limit problem that can be solved using the Squeeze Theorem, and know a strategy for applying the Squeeze Theorem to compute the limit.

(3) Given a limit, identify the function \(f(x)\) and value \(x = a\) that would make the limit represent the derivative of \(f(x)\) and \(x = a\). (You will NOT be asked to do this if it is impossible).

(4) Given a position function \(p(t)\), find the velocity function \(v(t)\), and acceleration function \(a(t)\) in terms of first and second derivatives, and answer questions posed in a word problem using these things.

(5) Know how to find the horizontal and vertical asymptotes of a function, if there are any.

(6) Know how to use the the IVT to solve a problem: this means word problems as well as questions about functions. Have a specific strategy in mind that works for you!

(7) Match the graph of a function to the graph of its derivative.

(8) Use a graph of a function \(f(x)\) to answer questions about its derivative without being given a formula for \(f'(x)\).

(9) Given a function \(f(x)\), find the formula for \(f'(x)\) using the limit definition of a derivative.

(10) Find the equation of the tangent line to a curve defined by a function at a given point using the derivative of the function.

(11) Identify the domain of a function we have worked with: trigonometric, exponential, polynomial, rational, absolute value function, greatest integer function, piecewise defined functions, and so on (any type of function you’ve seen in this course). You’re responsible for this skill in general as a precalculus skill.

(12) Find the relationship (or lack thereof) between the domain of a function \(f(x)\) and the domain of its derivative \(f'(x)\).

(13) Identify when algebra will make your calculus problem easier (before you perform the required computation), and when it won’t.

(14) Construct problem-specific strategies from scratch based on your Knowledge of the course material.

(15) Use all the rules from Sections 3.1 and 3.2 to differentiate functions (Derivatives of Constants, The General Power Rule, the Constant Multiple Rule, the Sum and Difference Rules, and the Derivative of an Exponential Function, the Product Rule, and the Quotient Rule).

(16) Given a function \(f(x)\), compute \(\lim_{x \to \infty} f(x)\) or \(\lim_{x \to -\infty} f(x)\), if the limit is a finite number. Know when \(\lim_{x \to \infty} f(x)\) or \(\lim_{x \to -\infty} f(x)\) is either \(+\infty\) or \(-\infty\).

(17) Given a formula for a function \(f(x)\) being differentiable at \(x = a\), find the equation of the tangent line to \(f(x)\) at \(x = a\) in \(y = mx + b\) form.

4. Skills for Exam 2

(1) Use implicit differentiation to find the tangent line to a curve that is implicitly defined (such as \(y^2 = \cos(x + y)\), or a point on an ellipse...).

(2) Use information about a function (asymptotes, first and second derivatives, local optima,...) to draw and identify graphs of a function as in problems taken from 4.3. See example 8 on page 296, and Worksheets 14 and 15 for examples of problems requiring these skills.

(3) Identify situations where it is necessary to use L’Hospital’s rule, and use it effectively to compute a limit.

(4) Apply tricks and strategies for different types of indeterminate forms when using L’Hospital’s rule.
(5) Identify which local and global maxima and minima are relevant in a word problem.
(6) Use the First and Second Derivative Tests to find local maxima and minima of a function.
(7) Use the second derivative to find intervals of concavity and inflection points.
(8) Understand when it is appropriate to apply the Closed Interval Method (page 278).
(9) Given a problem where \( F(x) = f(g(x)), f(g(h(x)), f(g(h(k(x)))) \) and so on, find \( F'(x) \) using the Chain Rule.
(10) Draw pictures effectively to clarify your understanding of related rates and optimization problems, when possible or necessary.
(11) Given a function \( F(x) = f(x)^{g(x)} \), know how to find \( F'(x) \) using logarithmic differentiation.
(12) Given a function \( F(x) \), identify \( F(x) \) as \( f(g(x)), f(g(h(x)), \) and so on.
(13) Use the constraints, inherent relationships, and critical thinking in an optimization problem to reduce the number of variables to one, so that you can apply the techniques of Chapter 4 to maximize or minimize a function, and answer a word problem.
(14) Use geometric reasoning and critical thinking to relate quantities that are functions of time in related rates problems.
(15) Drawing effective diagrams for related rates and optimization problems to clarify what the problem is asking for.
(16) Distinguish between the various types of indeterminate forms to which L’Hospital’s rule apply.
(17) Know how to derive the formula for the derivative of an inverse trigonometric function using implicit differentiation (see page 213-214).
(18) Use geometric reasoning and critical thinking to derive constraints and identify the function to be optimized in an optimization problem.
(19) Using algebra skills and knowledge of domains and ranges of specific functions to find critical points of derivatives that are not polynomials.
(20) Know how to apply the Mean Value Theorem (page 285) to obtain information about a function from information about its derivative (Section 4.2).
(21) Recognize a word problem that can be solved using an exponential growth or decay model, and set up a model to answer the word problem.

5. Clarifying Notes

(1) Studying with friends is always a great idea, but you should check with the Instructor or TAs to be sure you know the correct answer to a question or the correct method for doing a problem when in doubt.
(2) You do not need to memorize the Mean Value Theorem, Fermat’s Theorem, or Newton’s Law of Cooling for Exam 2.
(3) You will be supplied with a list of formulae for volumes, areas, and surface areas of common geometric objects on the exam. I will not tell you in advance what the formulae will be, but there will be no problems on the test that require a formula I do not provide.
(4) You not be tested past 4.7 on Exam 2. See Course schedule for which sections you’ll be tested on.
(5) While 3.3 and 4.5 are not listed as sections you’ll be tested on, you must be able to take trigonometric derivatives for Exam 2, you must know the trig circle of life and basic facts about trig functions, and you must be able to draw or identify the graph of a function with asymptotes, points of inflection, and local and global maxima and minima as we did in problems similar to problems from 4.3. See Knowledge and Skills above. These sections are not listed as tested because the problems from Webassign and the book from 3.3. and 4.5 are not representative of what you’ll be expected to do on the test.
(6) If you DO like to memorize things, we will not punish you on the test for using a memorized trigonometric or inverse trigonometric derivative when we grade, but be careful to still show work and justify answers sufficiently when you do.

(7) Please review the Exams part of the syllabus as well as the Test Taking Strategies on the Course Webpage.

(8) **DO NOT forget to bring at least ONE form of acceptable ID (as outlined in the syllabus) to Exam 2. Without this you cannot take the exam**-we will do ID checks

(9) Use your resources. These include: the Worksheet and Quiz Answers on the course website, the Webassign answers for problems on assignments that are due, videos in the e-book, the tutoring lab and my office hours—including the special morning office hour 10:30 the day of the exam—to make sure you know all the things in this guide, or ask about previous Webassign, Quiz, and Worksheet Problems.