Dear Lecture EL1, Math 221.

1. Correction to Rule 1 for Limits that I put on the board

Rule 1 is only true if the function $f$ is continuous. We haven’t defined the the word continuous yet, so don’t worry too much about that. It just means that nothing what is shown in the picture happens:

![Graph of a function](image)

We’ll get to continuity in Section 2.5.

2. Correction to Squeeze Theorem Problem

I made a serious type-O during lecture today in my squeeze theorem problem! The correct problem statement was

**Example**

Find the Limit:

$$\lim_{x \to 0} 2x^4 \sin \left( \frac{1}{x} \right)$$

What I actually wrote was:

$$\lim_{x \to 0} 2x^4 \left( \frac{1}{\sin x} \right)$$

Note:

$$\sin \left( \frac{1}{x} \right) \neq \left( \frac{1}{\sin x} \right)$$

And the problem is that since $\sin x = 0$ at any multiple of $\pi : (..., -\pi, 0, \pi, 2\pi ...)$ so we actually have the fact that

$$-\infty \leq \frac{1}{\sin x} \leq \infty$$

because as $\sin x$ gets close to zero, $\left| \frac{1}{\sin x} \right|$ approaches infinity.

With that out of the way, here’s the problem rewritten properly:

**Solution**

For all $y \in (-\infty, \infty)$, $-1 \leq \sin(y) \leq 1$. So, if we substitute $y = \frac{1}{x}$, this is still true no matter how close $x$ gets to 0.

We can write
\[-1 \leq \sin \left( \frac{1}{x} \right) \leq 1\]

So that

\[(2x^4)(-1) \leq 2x^4 \sin \left( \frac{1}{x} \right) \leq (2x^4)(1)\]

and so

\[
\lim_{x \to 0} -2x^4 \leq \lim_{x \to 0} 2x^4 \sin \left( \frac{1}{x} \right) \leq \lim_{x \to 0} 2x^4
\]

Now, 0 is in the domain of \(-2x^4\) and \(2x^4\), they are just polynomials, so by the Squeeze Theorem plug in \(x = 0\) to obtain

\[
0 \leq \lim_{x \to 0} 2x^4 \sin \left( \frac{1}{x} \right) \leq 0
\]

and we see that \(\lim_{x \to 0} 2x^4 \sin \left( \frac{1}{x} \right)\)

Deepest apologies about this typo! Feel free to interrupt me in lecture when you don’t understand or feel I’m making a typo. And, to finish the example from the end in more detail....

**Question 2.1.** What about

\[
\lim_{x \to 0} 2(x + 1) \sin \left( \frac{1}{x} \right)
\]

It is still true that

\[-1 \leq \sin \left( \frac{1}{x} \right) \leq 1\]

and it is also true that

\[2(x + 1)(-1) \leq 2(x + 1) \sin \left( \frac{1}{x} \right) \leq 2(x + 1)(1)\]

And you can check by plotting these functions in Wolfram Alpha or your favorite tool that near 0 that all the hypotheses of the Squeeze Theorem are satisfied near 0: in particular

\[-2x - 2 \leq 2(x + 1) \sin \left( \frac{1}{x} \right) \leq 2x + 2\]

So that (simplifying the polynomial expressions in the lower and upper bounds)

\[
\lim_{x \to 0} -2x - 2 \leq \lim_{x \to 0} 2(x + 1) \sin \left( \frac{1}{x} \right) \leq \lim_{x \to 0} 2x + 2
\]

\[
\lim_{x \to 0} -2x - 2 \leq \lim_{x \to 0} 2(x + 1) \sin \left( \frac{1}{x} \right) \leq \lim_{x \to 0} 2x + 2
\]

All we know is that

\[-2 \leq \lim_{x \to 0} 2(x + 1) \sin \left( \frac{1}{x} \right) \leq 2\]

And we just have an upper and lower bound on the limit, if it even exists.....