1. Estimate, up to an error of size $O(\epsilon^2)$, the eigenvalues and eigenvectors of
   
   (a) \[
   \begin{pmatrix}
   1 & \epsilon \\
   0 & 2
   \end{pmatrix},
   \]
   
   (b) \[
   \begin{pmatrix}
   1 & \epsilon & 2\epsilon \\
   0 & 2 & 0 \\
   1 - \epsilon & 0 & -1
   \end{pmatrix}.
   \]
   
   (c) \[
   \begin{pmatrix}
   \epsilon & 0 & 0 \\
   0 & 2\epsilon & 0 \\
   0 & 0 & 3\epsilon
   \end{pmatrix}.
   \]

2. Find two linearly independent solutions of
   \[u'' + q(x)u = 0,\]
   where
   \[q(x) = \begin{cases} 
   -1, & x < 0, \\
   1 & x > 0
   \end{cases}\]
   for $u$ defined on $x \in (-\infty, \infty)$. (This is problem 1 in Section 7.5 of Keener.)

3. Define the ODE by
   \[x' = 2x + \epsilon f(x, y), \quad y' = -3y + \epsilon g(x, y).\]
   Describe all possible monomial choices of $f$ and $g$ which can be removed by a near-identity change of coordinates. Compute what the change of coordinates would be in each case. Compute the system in the new coordinates in each case.

4. Define $f(u) = u - u^3/3 + 0.1$. First show that $F(u)$ defined by $F' = -f$ is a double-well potential and determine the relative depth of each well with respect to the other. Define the minima of these wells as $u_{\pm}$ as in class. Consider the boundary-value problem
   \[u'' + f(u) = 0\]
   with
   \[
   \lim_{x \to -\infty} u(x) = u_{-}, \\
   \lim_{x \to +\infty} u(x) = u_{+}.
   \]
   Describe why this problem does not have a solution using phase-plane arguments. Now consider the modified equation
   \[u'' + cu' + f(u) = 0\]
   with the same boundary conditions. Determine to four digits’ accuracy the value of $c$ which does give a solution to this boundary value problem.

   **Hint.** One could write original code to solve this problem, but you might find it easier to use code available on the web. Go to [http://math.rice.edu/~dfield/dfpp.html](http://math.rice.edu/~dfield/dfpp.html) and use the “pplane” software: this software allows you to type in a vector-field and plot trajectories by pointing. By playing with the value of $c$, you can determine which value of $c$ gives a solution to this problem to any desired accuracy.