1. Show that the fundamental solution $\psi(r)$,

$$\psi(r) = \begin{cases} 
  c_1 + c_2 \log r, & n = 2, \\
  c_1 + c_2 r^{2-n}, & n \geq 3,
\end{cases}$$

is integrable near the origin. Deduce from this that $\psi \in L^1_{\text{loc}}(\mathbb{R}^n)$.

2. (McOwen 4.2.1).

3. (McOwen 4.2.10).

4. Here we show that the Laplacian operator is also radially symmetric, or, if we consider any unitary transformation of $\mathbb{R}^n$, the Laplacian operator remains unchanged. First, we say that an $n \times n$ matrix $U$ is unitary if $UU^t = I$.

(a) Show that if $U$ is unitary, then $\|Ux\| = \|x\|$ for all $x \in \mathbb{R}^n$.

(b) Let $y = Ux$, where $U$ is unitary. Define

$$\Delta_x u = \sum_{i=1}^n \frac{\partial^2 u}{\partial x_i^2}, \quad \Delta_y u = \sum_{i=1}^n \frac{\partial^2 u}{\partial y_i^2}.$$  

Show that for all $u \in C^2(\mathbb{R}^n)$, $\Delta_x u = \Delta_y u$.

5. (McOwen 4.4.2).

6. (McOwen 4.4.6).