1. **(Strauss 5.2.2.)** Show that $\cos(x) + \cos(\alpha x)$ is periodic if $\alpha$ is a rational number and compute its period. What happens if $\alpha$ is not rational?

2. Define $f(x) = x^3$ on the interval $[0, 1]$. Compute its Fourier sine series and its Fourier cosine series.

3. Consider the function $f(x) = x$ on the interval $[-\pi, \pi]$. Compute the full Fourier series for $f(x)$. Use Parseval’s Identity to compute $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

4. Solve the heat equation given by
   \[ u_t = ku_{xx}, \quad x \in [0, L], \ t > 0, \]
   \[ u(x, 0) = x, \]
   \[ u(0, t) = u(L, t) = 0. \]

5. **(Strauss 5.3.8.)** Let $f$ and $g$ satisfy the same Robin boundary condition at $x = 0$ and the same Robin boundary condition at $x = L$ (i.e., we assume that
   \[ f'(0) + \alpha f(0) = g'(0) + \alpha g(0) = f'(L) + \beta f(L) = g'(L) + \beta g(L) = 0. \]
   Prove then that
   \[ (f'(x)g(x) - f(x)g'(x))|_{x=L}^{x=0} = 0. \]
   Deduce from this that eigenfunctions of a Robin BVP are orthogonal.

6. Prove that if $f$ has period $p$, then
   \[ \int_{p+a}^{a} f(y) \, dy \]
   is independent of $a$.

7. Consider the infinite list of functions
   \[ \{1, \cos(x), \cos(2x), \ldots, \cos(nx), \ldots, \sin(x), \sin(2x), \ldots, \sin(nx), \ldots\}. \]
   Show that this is an orthogonal set of functions on the set $[-\pi, \pi]$, i.e. if we define the inner product
   \[ \langle f, g \rangle := \int_{-\pi}^{\pi} f(x)g(x) \, dx, \]
   then if we choose any two different functions from that list, then their inner product is zero.

8. Let $\{f_n(x)\}$ be any sequence of functions which converge to $f(x)$ uniformly on $[a, b]$. Prove then that $f_n(x)$ converge to $f$ in the $L^2$ sense as well. Show a counterexample to demonstrate that the converse is false, i.e. that we can have $L^2$ converge but not uniform. (The term used for this is that uniform convergence is stronger than $L^2$).