1. **1.11.** It does not matter. The best way to see this is to note that applying a 15% discount to an item is the same as multiplying its price by 0.85, and applying 5% tax to an item is the same as multiplying its price by 1.05.

The computation by law is then $1.05 \times (0.85 \times x)$, where $x$ is the price. The customer wants to compute $0.85 \times (1.05 \times x)$ instead. However, using both the associative and commutative laws of multiplication, these are the same.

2. **1.13.** We define

\[ A = \{ n \in \mathbb{Z} : n = 2k + 1 \text{ for some } k \in \mathbb{Z} \}, \]
\[ B = \{ n \in \mathbb{Z} : n = 2k - 1 \text{ for some } k \in \mathbb{Z} \}. \]

We want to show that $A = B$.

First we show $A \subseteq B$. The way to do this is to pick an arbitrary element in $A$ and show it is also in $B$. Thus, choose $n \in A$. By definition, $n = 2k + 1$, where $k \in \mathbb{Z}$. We can rewrite $n = 2k + 1 = 2(k + 1) - 1$. Since $k + 1 \in \mathbb{Z}$ whenever $k$ is, we have $k + 1 \in \mathbb{Z}$, so by definition $2(k + 1) - 1 \in B$ and thus $n \in B$.

We now need to show $B \subseteq A$, but it is the same argument in reverse. Let $n \in B$, and thus $n = 2k - 1$ for some $k \in \mathbb{Z}$. We can again rewrite this as $n = 2k - 1 = 2(k - 1) + 1$. Since $k - 1 \in \mathbb{Z}$ whenever $k$ is, we have $k - 1 \in \mathbb{Z}$ and $n \in A$.

3. **1.18.** We are asking for the solutions of the equation $x = 1 + \frac{1}{x}$.

First notice that the right-hand side is not defined for $x = 0$, so we can assume that $x \neq 0$ in what follows. We multiply both sides of the equation by $x$, and collect all of the terms to the left-hand side, to obtain

\[ x^2 - x - 1 = 0. \]

(Since $x \neq 0$, multiplying both sides by $x$ does not change the number of solutions.) Using the quadratic formula, we have the solutions are

\[ x = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}. \]

So there are exactly two numbers which satisfy this:

\[ x = \frac{1 + \sqrt{5}}{2}, \quad \frac{1 - \sqrt{5}}{2}. \]

4. **1.25.** The key to this problem is to note that at the point in time where the census taker knows the product of the ages is 36, and the sum is some particular number (which we don’t know), he does not know what the ages of the daughters are. Let us first enumerate the possibilities, each of which I list from oldest to youngest:

<table>
<thead>
<tr>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>1</td>
<td>1</td>
<td>38</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>1</td>
<td>13</td>
</tr>
</tbody>
</table>

1
From this, we see that the house number must be 13. If it were anything else, then the census taker could have figured out the ages right then. So after the census taker asks for more information, we are told that there is an oldest daughter. This eliminates the (6, 6, 1) case and thus the daughters are (9, 2, 2).

5. 1.36. To prove \( S \subseteq T \), it suffices to go through each element in \( S \) and show that it also lies in \( T \). So, we have

\[
\begin{array}{|c|c|c|}
\hline
x & y & 3x + y - 4 \\
\hline
1 & 1 & 0 \\
1 & 2 & 1 \\
1 & 3 & 2 \\
2 & 1 & 3 \\
2 & 2 & 4 \\
2 & 3 & 5 \\
3 & 1 & 6 \\
3 & 2 & 7 \\
3 & 3 & 8 \\
\hline
\end{array}
\]

As we can see, for any \((x, y) \in S\), we also have \( 0 \leq 3x + y - 4 \leq 8 \), or \((x, y) \in T\).

Since \( S \subseteq T \), to show \( S = T \) we would need to show that \( T \subseteq S \). However, this is not true, and we can show this by picking any element of \( T \) not in \( S \). Notice that \((4, 0)\) and \((0, 4)\) are both in \( T \) and clearly not in \( S \). In fact, if we make \( x \) really large and positive, we can cancel with a large and negative \( y \), so that we could also pick \( x = 1,000,002, y = -3,000,000 \), and we would have \((x, y) \in T\). This sort of idea could show that there are many (in fact infinitely many) elements of \( T \) not in \( S \). But, of course, to show the two sets are not equal we need only one.

6. 1.42. No. How many days are there in February?

7. 1.49.

(a) True. The way to prove this is as follows. We know that the definition of bounded means that for all \( x \in \mathbb{R} \), there is an \( M \) such that

\[ |f(x)| < M \]

and there is an \( N \) such that

\[ |g(x)| < N. \]

Now, check:

\[ |(f + g)(x)| = |f(x) + g(x)| \leq |f(x)| + |g(x)| = M + N. \]

The first equality is by definition, and the \( \leq \) is by the Triangle Inequality.

(b) True. This proof has a similar setup, except the last line is

\[ |(fg)(x)| = |f(x)g(x)| = |f(x)||g(x)| = MN. \]

(c) False. Notice simply that if we choose \( g = -f \), then \( f + g = 0 \) for all \( x \), and this is certainly bounded. So, if we pick \( f(x) \) to be any unbounded function, and \( g(x) \) its negative, then we have shown a case where \( f + g \) is bounded, but \( f \) and \( g \) are both unbounded. So, for example, choose \( f(x) = x \) and \( g(x) = -x \).

(d) False. Similar idea to above, if we choose \( g = 1/f \) then \( fg = 1 \) for all \( x \), which is bounded. So, pick, for example, \( f(x) = x \) and \( g(x) = 1/x \). Both of these functions are unbounded, but their product is bounded.
(e) True. The way to think about this (this is not a proof!) is as follows: let’s say that we had an unbounded function $f$, and we wanted to generate a $g$ so that $f + g$ and $fg$ were bounded. Let’s say that $f(x)$ gets large and positive for some $x$. The way to cancel this, so as to make $f + g$ bounded, would be to make $g$ large and negative, but then this would make $fg$ large and negative. On the other hand, if we chose $g$ to make $fg$ small, then we would need $g$ very close to zero, but then this would make $f + g$ large.

Now, here is the proof. We first note that if a function $h$ is bounded, then so is $h^2$ (simply use part (b) with $f = g = h$). We also know that if we multiply a bounded function by a constant, then it is also bounded (use part (b) with $g$ a constant function). Then we have

$$(f + g)^2 = f^2 + 2fg + g^2.$$  

Since $f + g$ is bounded, so is $(f + g)^2$ and thus so is $f^2 + 2fg + g^2$. On the other hand, since $fg$ is bounded, so is $-2fg$. Then

$$f^2 + g^2 = (f + g)^2 - 2fg,$$

and thus we know $f^2 + g^2$ is bounded (use part (a)). But notice that

$$|f(x)|^2 = |f^2(x)| \leq |f^2(x) + g^2(x)|$$

for all $x$, and since $f^2 + g^2$ is bounded, we know that there is an $M$ such that

$$|f(x)|^2 \leq |f^2(x) + g^2(x)| < M,$$

or

$$|f(x)|^2 < M.$$  

However, this means then that $|f(x)| \leq \sqrt{M}$ and thus $f$ is bounded. (The same argument works for $g$.)