1. Identify the following integrals as improper or proper. If the integrals are improper, then indicate the point or points which cause the integral to be improper. You do not need to evaluate the integrals or to say if they converge or diverge.

(a) \( \int_{0}^{\infty} e^{-x} \, dx \)

(b) \( \int_{0}^{1} \frac{1}{1 - x^2} \, dx \)

(c) \( \int_{-\frac{1}{2}}^{7} \frac{1}{1 - x^3} \, dx \)

(d) \( \int_{0}^{\infty} \frac{2 + \cos(x)}{x^2 - 9x + 8} \, dx \)

2. You get to play TA for a day! One of your fellow calculus students was asked to evaluate the integral below. The student’s solution was the following:

\[
\int_{0}^{4} \frac{1}{\sqrt{4-x}} \, dx = -2\sqrt{4-x}\bigg|_{0}^{4} = -2\sqrt{4-4} + 2\sqrt{4} = 4
\]

Grade this student’s solution. Is it correct? If not, where does it go wrong and why is it incorrect?

3. As we saw in class, the region \( R = \{(x, y) : x \geq 1, 0 \leq y \leq 1/x\} \) has infinite area. The Horn of Gabriel is formed by rotating this region about the \( x \)-axis. Make a careful sketch of \( R \) and of the Horn. Then find the volume of the Horn of Gabriel.
4. Some improper integrals are given. For each one, either evaluate, or show that it diverges. First, identify if the integral is of type 1 or 2 and then write out the corresponding limit before you start to solve the integral. Do not calculate the integral before you have the correct limit.

(a) \( \int_{0}^{1} \ln x \, dx \)

(b) \( \int_{0}^{\infty} \sin x \, dx \)

(c) \( \int_{0}^{\frac{\pi}{2}} \tan x \, dx \)
(d) \( \int_0^\infty \frac{1}{1+x^2} \, dx \)

(e) \( \int_e^\infty \frac{1}{x \ln x} \, dx \)

(f) \( \int_0^\infty e^{-\frac{x}{2}} \, dx \)
5. Evaluate the following improper integrals.

(a) \[ \int_0^1 \frac{1}{x^{0.9}} \, dx \]

(b) \[ \int_0^1 \frac{1}{x} \, dx \]

(c) \[ \int_0^1 \frac{1}{x^{1.1}} \, dx \]

(d) Can you use your results above to hypothesize for which \( p > 0 \) is an improper integral of the form \( \int_0^1 \frac{1}{x^p} \, dx \) convergent?

6. Evaluate the following improper integrals.

(a) \[ \int_1^\infty \frac{1}{x^{0.9}} \, dx \]

(b) \[ \int_1^\infty \frac{1}{x} \, dx \]

(c) \[ \int_1^\infty \frac{1}{x^{1.1}} \, dx \]

(d) Can you use your results above to hypothesize for which \( p > 0 \) is an improper integral of the form \( \int_1^\infty \frac{1}{x^p} \, dx \) convergent?