In this worksheet we take derivatives and integrals of parametric equations:

1. Let $x = f(t)$ and $y = g(t)$ be a parametrization. What is the formula for the area under the curve in terms of $t$? (Hint: normally it should be $\int y \, dx$)

2. Consider the parametric curve $x = \sin^2 t$, $y = \sin 3t$, $0 \leq t \leq \pi/3$. Set up but do not evaluate integrals which represent the following:
   a) The area under the curve.
   b) The surface area created by rotating the curve about the $x$-axis.
   c) The surface area created by rotating the curve about the line $y = 5$.
   d) The surface area created by rotating the curve about the $y$-axis.
3. Let \( x = f(t) \) and \( y = g(t) \) be a parametrization. Find \( \frac{dy}{dx} \) in terms of \( f(t) \) and \( g(t) \).

4. Find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) in terms of \( t \):
   a) \( x = t^3 + t^2 + 1, \ y = 1 - t^2 \).
   b) \( x = 1 + t^2, \ y = t \ln t \).

In physics, the velocity of an particle which moves on the trajectory \( x = f(t) \) and \( y = g(t) \) is \( \left( \frac{dx}{dt}, \frac{dy}{dt} \right) \), but the speed is \( \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} \).

5. Assume an object moves on the path \( x = \cos(3t) \) and \( y = -\sin(3t) \). Calculate the velocity and speed of the object.

What can we conclude from this?
Let’s go back to power series for a little bit:

6. Given that the power series

\[ \sum_{n=0}^{\infty} c_n (x - a)^n \]

converges to \( f(x) \) for all \( x \), find a formula for

a) a power series which converges to \( f'(x) \).

b) a power series which converges to \( (x - a)^{-1}(f(x) - c_0) \)

c) a power series which converges to \( xf(x + a) \)

d) \( f'''(a) \)

e) a series which converges to the integral \( \int_{a}^{2a} f(x) \, dx \)
7. Interpret the integral $I = \int_0^1 \sqrt{1 + 4x^2} \, dx$ in two different ways and evaluate each way numerically:

   a) as the length of an appropriate curve.

   b) as the area of something.

8. a) Find the equation of an ellipsoid with radius on the $x$-axis length $a$ and radius on the $y$-axis length $b$.

b) Use that equation to find a parametrization of your equation.

c) Use that parametrization to find the circumference of the ellipse.

d) Now use it to find the area of an ellipse

e) An ellipsoid is the shape you get from rotation the ellipse around the $x$-axis. Use the parametrization to find the surface area of the ellipse.