This worksheet reviews material from Chapter 11, Infinite Sequences and Series.

1. a) Give an example of (i) a divergent sequence, (ii) a convergent sequence.

b) Give an example of (i) a divergent series, (ii) an absolutely convergent series, (iii) a conditionally convergent series.

2. True or false (if false give a counter example):

   a) If \( \{a_n\} \) is convergent then \( \sum_{n=0}^{\infty} a_n \) is convergent.

   b) If \( \sum_{n=0}^{\infty} a_n \) is convergent then \( \{a_n\} \) is convergent.

   c) If \( \{a_n\} \) is convergent then \( \sum_{n=0}^{\infty} (a_n - a_{n+1}) \) is convergent.

   d) If \( \sum_{n=0}^{\infty} a_n \) is convergent, then \( \sum_{n=0}^{\infty} (-1)^n a_n \) is also convergent.

   e) If \( \sum_{n=0}^{\infty} a_n \) is convergent, then \( \sum_{n=0}^{\infty} (-1)^n a_n \) is absolutely convergent.
3. Determine whether the series \( \sum_{n=2}^{\infty} \frac{(\ln n)^4 (-1)^{n+1}}{n + 2} \) converges conditionally, converges absolutely, or diverges.

4. Consider the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 3}{n^5} \). The first 10 terms are used to approximate the sum. According to the alternating series estimation theorem, what is the maximum error in this approximation?

5. What can we say about the absolute and conditional convergence of \( \sum_{n=1}^{\infty} (-1)^n \frac{n!}{n^n} \)?
6. What is the difference between Taylor series and a MacLaurin series?

7. Find a power series expansion, centered at 0, for \( \frac{\cos(x^3) - 1}{x^2} \). Determine the interval of convergence.

8. Use that to find \( \lim_{x \to 0} \frac{\cos(x^3) - 1}{x^2} \):

9. Find the exact value of \( 1 - \frac{\pi^2}{2^2 \cdot 2!} + \frac{\pi^4}{2^4 \cdot 4!} - \frac{\pi^6}{2^6 \cdot 6!} + \cdots \).
Couple (all over the place) questions:

10. What is the limit of the sequence: 0.9, 0.99, 0.999, ... ? (Actually show it)

11. Let \( a_n = \left(\frac{1}{2}\right)^2 \left(\frac{2}{3}\right)^3 \cdots \left(\frac{n}{n+1}\right)^{n+1} \) (in particular \( a_1 = \frac{1}{4}, a_2 = \frac{1}{4} \cdot \frac{8}{27} = \frac{2}{27} \)). Find \( \lim_{n \to \infty} a_n \).

12. Let \( m \) be a natural number. Propose a method to find the precise value for \( \sum_{n=0}^{\infty} \frac{1}{n!(n + m + 1)} \).