1. Write a precise definition of the Taylor series and the MacLaurin series.

2. a) Find the MacLaurin series of \( \frac{1}{1+x} \)

b) Use that to find the MacLaurin series of \( \ln(x + 1) \)

c) Find the MacLaurin series of \( \frac{1}{(1+x)^2} \)

3. Find the MacLaurin series of \( \tan^{-1}(x) \)
We use Taylor series to calculate integrals of functions we normally cannot calculate

4. Augustin-Jean Fresnel (1788-1827) was an engineer, mathematician and the French commissioner of lighthouses. He is famous for his work in optics and for developing the Fresnel lens. Originally developed for lighthouses, Fresnel lenses are still used today in many consumer items including computer and overhead projectors. The integral

$$\int_0^1 \frac{\sin(x)}{x} \, dx$$

occurs in Fresnel’s theory of diffraction, and is known as a Fresnel integral.

(a) Use the Maclaurin series for $\sin(x)$ to evaluate the Fresnel integral as an infinite series.

(b) How many terms do you have to add to get within $10^{-3}$ of the answer?

5. (a) Use the Maclaurin series for $\cos x$ to find the Maclaurin series for $f(x) = x^3 \cos(x^2)$.

(b) Use part (a) to find the value of $f^{(11)}(0)$. Hint: this should be easy.
6. Find the Taylor series (centered at the given point) for each of the following functions by differentiating the given functions and finding the pattern.
(a) \( f(x) = \sin x \) centered at \( a = \frac{\pi}{2} \).

(b) \( g(x) = \ln x \) centered at \( a = 2 \).

7. Use power series to find \( \int e^{x^2} \, dx \)

8. a) Write down the first three terms of the Maclaurin series for \( f(x) = \frac{\sin(x^2) - x^2 \cos x}{x^4} \).

b) Use this to evaluate \( \lim_{x \to 0} f(x) \).
One of the great things about power series is that we can use them to calculate the values of certain series:

9. Find the value of the following series:

a) \(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\)

b) \(\frac{\pi^2}{2!} - \frac{\pi^3}{3!} + \frac{\pi^4}{4!} - \frac{\pi^5}{5!} + \ldots\)

c) \(2 - \frac{8}{3} + \frac{32}{5} - \frac{128}{7} + \ldots\) (maybe \(\arctan(x)\) can help you)