1. Let $\sum_{n=0}^{\infty} (-1)^n a_n$ be a series. State three different tests (with their conditions), which tell us about (absolute) convergence of the series.

**Proof.** There are several. Here are three cases:

1. Ratio test: Let $l = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$. If $l > 1$ then the series is absolutely convergent, if $l < 1$ then it is divergent and if $l = 1$ then we can’t make any conclusion.

2. Root test: Let $l = \lim_{n \to \infty} \sqrt[n]{|a_n|}$. If $l > 1$ then the series is absolutely convergent, if $l < 1$ then it is divergent and if $l = 1$ then we can’t make any conclusion.

3. Alternating series test: If the sequence $\{a_n\}$ is positive, decreasing and $\lim_{n \to \infty} a_n = 0$, then the series is conditionally convergent.

2. Find the McLaurin series of $\sin(x) \cos(x)$ (writing it as a product of two power series is not an answer).

**Proof.** Remember $\sin(x) \cos(x) = \frac{1}{2} \sin(2x)$. So:

$$
\sin(x) \cos(x) = \frac{1}{2} \sin(2x) = \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!} = \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \frac{(2)^{2n+1}}{(2n+1)!} x^{2n+1}
$$

3. Find $\int_0^1 x^3 \arctan(x^2) \, dx$.

**Proof.** Remember the power series of $\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$.

So:

$$
x^3 \arctan(x^2) = x^3 \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+5}}{2n+1}
$$

Now we can compute the integral:

$$
\int_0^1 x^3 \arctan(x^2) \, dx = \int_0^1 \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+5}}{2n+1} \, dx = \sum_{n=0}^{\infty} \int_0^1 (-1)^n \frac{x^{4n+5}}{2n+1} \, dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(4n+6)}
$$