Study Goals: • Use trig substitutions to solve some (tricky!) integrals.

1. First we have to remind ourselves of basic facts about trig functions:
It is important to understand the connection between different trig functions (drawing triangles can help a lot).

1. Let \( x = 3 \sin \theta \). Express the six major trig functions of \( \theta \) in terms of \( x \).
   \[
   \sin \theta = \ldots \quad \cos \theta = \ldots \\
   \sec \theta = \ldots \quad \csc \theta = \ldots \\
   \tan \theta = \ldots \quad \cot \theta = \ldots 
   \]

2. Do the same for when \( x = 3 \tan \theta \).

3. Do the same for when \( x = 3 \sec \theta \).

2. Fill in the table.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Substitution</th>
<th>( dx )</th>
<th>Identity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{a^2 - x^2} )</td>
<td>( x = a \sin \theta )</td>
<td>( dx = a \cos \theta d\theta )</td>
<td>( a^2 - a^2 \sin^2 \theta = a^2 \cos^2 \theta )</td>
</tr>
<tr>
<td>( \sqrt{a^2 + x^2} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sqrt{x^2 - a^2} )</td>
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</tbody>
</table>
3. Evaluate the integrals using trigonometric substitution. State the necessary restriction on the angle $\theta$.

(a) $\int \frac{x^2}{\sqrt{9-x^2}} \, dx$

(b) $\int \frac{1}{\sqrt{25+x^2}} \, dx$

4. Evaluate $\int \frac{x^3}{\sqrt{x^2-9}} \, dx$.

**Hint:** Instead of trigonometric substitution, try substituting $u = \sqrt{x^2-9}$. This trick would also work on $\int \frac{x}{\sqrt{x^2-9}} \, dx$, but would not work on $\int \frac{x^2}{\sqrt{x^2-9}} \, dx$ or $\int \frac{x^4}{\sqrt{x^2-9}} \, dx$.
Sometimes, you first have to complete a square and then use trig substitution, so first let’s practice that:

5. Complete the following squares:
   (a) \(-x^2 + 4x = -(_____ - _____)^2 + ______

   (b) \(x^2 - 4x + 20\)

Now use your square completion abilities to solve the next ones:

6. Evaluate \(\int \frac{1}{\sqrt{x^2 + 2x}} \, dx\).

7. Evaluate \(\int ((x - 2)^3)\sqrt{5 + 4x - x^2} \, dx\).
You have to be careful. Sometimes trig substitution makes the question more complicated.

8. Evaluate \( \int \frac{x^2}{\sqrt{x^2 - 9}} \, dx \).

Let’s do another fun problem:

\[
\int \frac{4}{(16 + x^2)^2} \, dx
\]