Study Goals:

- Define parametric equations
- Draw graphs using parametric equations

1. Let \( x = f(t) \) and \( y = g(t) \) be a parametrization.

   - Find the formula for the area under the curve in terms of \( t \)
     (Hint: normally it should be \( \int y \, dx \))

   - Find the formula for the surface area created by rotating the curve about the \( x \)-axis in terms of \( t \) (Hint: normally it should be \( 2\pi \int y \, ds \))

2. Consider the parametric curve \( x = \sin^2 t, \ y = \sin 3t, \ 0 \leq t \leq \pi/3 \). Set up but do not evaluate integrals which represent the following:

   a) The area under the curve.

   b) The arc length on the given interval

   c) The surface area created by rotating the curve about the \( x \)-axis.

   d) The surface area created by rotating the curve about the line \( y = 5 \).

   e) The surface area created by rotating the curve about the \( y \)-axis.
3. A sphere of radius $r$ is formed by rotating the semicircle

$$x = r \cos \theta, \quad y = r \sin \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

about the $y$ axis. Sketch a graph. Then compute the surface area of the sphere. (The answer should be familiar)

**Moral:** We can use parametric equations to compute arc length, surface area, ... of many shapes.

**Polar Coordinates:** Polar coordinates allow us to think of points $(x, y)$ in terms of $(r, \theta)$ (radius, angle).

4. Fill in the blanks:
If we have a radius $r$ and angle $\theta$ then we can find the $(x, y)$-coordinates by:

$$x = \_ \cos(\theta)$$
$$y = r \_$$

On the other hand if we have $(x, y)$-coordinates, we find the radius and angle by:

$$\theta = \arctan\left( \frac{y}{\_} \right) \quad \text{or} \quad \tan(\theta) = \_$$
$$r = \sqrt{x^2 + (\_)^2} \quad \text{or} \quad r^2 = \_$$

Although we can translate between $(x, y)$ and $(r, \theta)$ we must learn to directly think in terms of polar coordinates without any reference to $(x, y)$ coordinates.

5. Sketch the regions
   a) $1 \leq r \leq 2, \ -\frac{\pi}{3} \leq \theta \leq \frac{\pi}{4}.$
   b) $r \leq 0, \ \frac{4\pi}{3} \leq \theta \leq \frac{5\pi}{3}.$
6. Find the following polar coordinates on the plane

\((-2, \pi/4)\) \((\pi, 0)\) \((2, \pi/4)\) \((2, 9\pi/4)\) 
\((-2, 7\pi/4)\) \((0, 1)\) \((1, 3\pi/4)\) \((-1, 3\pi/4)\)

7. Identify each polar curve by finding a Cartesian equation.
   a) \(\theta = \frac{\pi}{3}\)
   
   b) \(r = 4\)
   
   c) \(r = 2\sin \theta\)

8. Find a simple polar equation which represents each of the following.
   a) \(x = 4\)
   
   b) \(y = 3x\)
   
   c) \(y = 4x^2\)
   
   d) \(x^2 - y^2 = 1\)
Why do we use polar coordinates? Solve the following problem to find out:

9. Find the equation of a circle of radius 1 and center (0, 0):
   a) Using $(x, y)$ coordinates
   b) Using polar coordinates

   c) What do you conclude from the previous two parts? In particular, answer following questions:
      I) Which one is a function?
      II) How do the graphs of part (a) and (b) compare?
      III) How do the formulas of part (a) and (b) compare?

10. Graph the following polar functions
   a) $r = \theta$
   b) $f(\theta) = \cos \theta$