Learning Objective: Win!

0) First Round

0) Find the surface area of shape we get by rotating $\frac{1}{x}$ around the $x$ axis (where $1 \leq x < +\infty$). (3 Points)

• Answer: $\pi$

0) Solve the integral $\int_{0}^{\pi/2} \cos^2(t) \sin(t) \, dt$ (3 Points)

• Answer: $\frac{1}{3}$

0) Find the degree 4 Taylor polynomial for $f(x) = x^2 \cos\left(\frac{x}{2}\right)$ at the point 0. (3 Points)

• Answer: $f(x) = x^2 - \frac{x^4}{4}$

0) Convert the equation $y^2 = y - x^2$ to $(r, \theta)$ coordinates (3 Points)

• Answer: $r = \sin(\theta)$

0) Does the series $\sum_{n=2}^{\infty} \frac{1}{n} - \frac{1}{n+1}$ converge or diverge (3 Points)

• Answer: Converges

1) Integration by Parts

1) $\int x \cos x \, dx$ (1 Point)

• Answer: $x \sin(x) + \cos(x) + c$

1) $\int \ln(x) \, dx$ (2 Points)

• Answer: $x \ln(x) - x + c$

1) $\int x^4 \ln x \, dx$ (3 Points)

• Answer: $\frac{x^5 \ln(x)}{5} - \frac{x^5}{25} + c$

1) $\int x^2 e^x \, dx$ (4 Points)

• Answer: $x^2 e^x - 2xe^x + 2e^x + c$
1) \( \int e^x \sin(2x) \, dx \) (5 Points)

- **Answer:** \( \frac{1}{5} e^x \sin(2x) - \frac{2}{5} e^x \cos(2x) + c \)

2) Trig Integrals Trig Substitution

2) \( \int \tan^3 x \sec^3 x \, dx \) (1 Point)

- **Answer:** \( \frac{1}{5} \sec^5(x) - \frac{1}{3} \sec^3(x) + c \)

2) \( \int \sin^2 x \cos^3 x \, dx \) (2 Points)

- **Answer:** \( \frac{1}{3} \sin^3(x) - \frac{1}{5} \sin^5(x) + c \)

2) \( \int \frac{1}{\sqrt{x^2 - 7}} \, dx \) (3 Points)

- **Answer:** \( \ln(\sqrt{x^2 - 7} + x) \)

2) \( \int \frac{4}{(16 + x^2)^2} \, dx \) (4 Points)

- **Answer:** \( \frac{1}{32} \left( \frac{4x}{x^2 + 16} + \tan^{-1} \left( \frac{x}{4} \right) \right) + c \)

2) \( \int \sin^2 x \cos^2 x \, dx \) (5 Points)

- **Answer:** \( \frac{x}{16} - \frac{\sin(4x)}{64} + \frac{\sin^3(2x)}{48} \)

3) Partial Fractions

3) Find the partial fraction decomposition of \( \frac{1}{x^2} \). (1 Point)

- **Answer:** \( \frac{A}{x} + \frac{B}{x^2} \)

3) Find the partial fraction decomposition of \( \frac{1}{x^3 + x^2 - 2x} \). (2 Points)

- **Answer:** \( \frac{A}{x} + \frac{B}{x + 2} + \frac{C}{x - 1} \)

3) \( \int \frac{7}{x^2 + 3x - 10} \, dx \) (3 Points)

- **Answer:** \( \ln(2 - x) - \ln(x + 5) + c \)
3) \( \int \frac{x^5 + 2}{x^2 - 1} \, dx \) (4 Points)

- **Answer:** \( \frac{1}{4}x^4 + \frac{1}{2}x^2 + 6 \ln(1 - x) - 2 \ln(x + 1) \)

3) \( \int \frac{x + 2}{(x - 1)(x - 2)^2} \, dx \) (5 Points)

- **Answer:** \( -\frac{4}{x - 2} - 3 \ln(x - 2) + 3 \ln(x - 1) + c \)

4) Arc Length & Surface Area

4) Find the arc length of \( y = 2x \) on the interval \([0, 3]\) (1 Point)

- **Answer:** \( 3\sqrt{5} \)

4) Set up the integral for surface area of \( x = t, \ y = t^2; -1 \leq t \leq 1 \) around \( y = 5 \) (2 Points)

- **Answer:** \( \pi \int_{-1}^{1} (5 - t^2)^2 \sqrt{1 + 4t^2} \, dt \)

4) Find the arc length of \( y = \sqrt{9 - x^2} \) on the interval \([-3, 3]\) (3 Points)

- **Answer:** \( 3\pi \)

4) Find the area under the curve \( x = t^3, \ y = 2t^2 + 1; -1 \leq t \leq 1 \) (4 Points)

- **Answer:** \( \frac{22}{5} \)

4) Find the arc length of \( x = e^t \sin t, \ y = e^t \cos t; 0 \leq t \leq \pi \) (5 Points)

- **Answer:** \( e^\pi - 1 \)

5) Integral Test

5) Does \( \sum_{n=2}^{\infty} \frac{3n - 1}{4n + 5} \) converge or diverge? (1 Point)

- **Answer:** Diverge

5) Does \( \sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2} \) converge or diverge? (2 Points)

- **Answer:** Converge

5) Does \( \sum_{n=2}^{\infty} \frac{1}{n(\ln(n)^2 + \ln(n) - 3)} \) converge or diverge? (3 Points)

- **Answer:** Converge
5) Does \( \sum_{n=4}^{\infty} \frac{1}{n \ln(n)(\ln(\ln(n)))} \) converge or diverge? (4 Points)

• \textbf{Answer: Diverge}

5) How many terms of \( \sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2} \) you have to add to estimate it within 0.01? (5 Points)

• \textbf{Answer: } n > e^{100}

6) Comparison Test & Limit Comparison Test

6) Does \( \sum_{n=1}^{\infty} \frac{n}{n^2 + 1} \) converge or diverge? (1 Point)

• \textbf{Answer: Diverge}

6) Does \( \sum_{n=1}^{\infty} \frac{n^2 + 3n^6 - 7n}{n^4 + 2n^8 - 6n^2} \) converge or diverge? (2 Points)

• \textbf{Answer: Converge}

6) Does \( \sum_{n=1}^{\infty} \frac{\ln(n)}{n} \) converge or diverge? (3 Points)

• \textbf{Answer: Diverge}

6) Does \( \sum_{n=1}^{\infty} \frac{\ln(n)}{n^3} \) converge or diverge? (4 Points)

• \textbf{Answer: Converge}

6) Does \( \sum_{n=1}^{\infty} \sin\left(\frac{\pi}{n}\right) \) converge or diverge? (5 Points)

• \textbf{Answer: Diverge}

7) Alternating Series

7) Does \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \) converge absolutely, conditionally or diverge? (1 Point)

• \textbf{Answer: Converge Conditionally}

7) Does \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n - 10)}{n^2 + 2} \) converge absolutely, conditionally or diverge? (2 Points)

• \textbf{Answer: Converge Conditionally}

7) Does \( \sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n^2 + 1}}{\ln n} \) converge absolutely, conditionally or diverge? (3 Points)
7) Does \( \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos n}{n^2} \) converge absolutely, conditionally or diverge? (4 Points)

- **Answer:** **Diverge**

7) How many terms of \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \) do we have to add to get 2 decimal places? (5 Points)

- **Answer:** \( n = 5 \)

8) Root & Ratio Test

8) Does the series \( \sum_{n=1}^{\infty} \frac{1}{(2n)!} \) absolutely converge or diverge? (1 Point)

- **Answer:** **Converges Absolutely**

8) Does the series \( \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{5^n(n+1)} \) absolutely converge or diverge? (2 Points)

- **Answer:** **Converges Absolutely**

8) Does the series \( \sum_{n=1}^{\infty} \frac{3^n}{\ln n} \) absolutely converge or diverge? (3 Points)

- **Answer:** **Diverges**

8) Does the series \( \sum_{n=0}^{\infty} \left[ \frac{2}{(-1)^n - 3n} \right]^n \) absolutely converge or diverge? (4 Points)

- **Answer:** **Diverges**

8) Does the series \( \sum_{n=0}^{\infty} 2^{(-1)^n-n} \) absolutely converge or diverge? (5 Points)

- **Answer:** **Converge Absolutely**

9) Finding Taylor Series

9) Find the McLaurin series of \( \frac{x}{1-x} \) (1 Point)

- **Answer:** \( \sum_{n=0}^{\infty} x^{n+1} \)

9) Find the Taylor series around 1 of \( 1 + 2x \) (2 Points)

- **Answer:** \( 3 + 2(x - 1) \)
9) Find the McLaurin series of \( \frac{x^2}{(1-x)^2} \) (3 Points)

- **Answer:** \( \sum_{n=0}^{\infty} (n+1)x^{n+2} \)

9) Find the McLaurin series of \( x^2 \arctan(x^3) \) (4 Points)

- **Answer:** \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+5}}{2n+1} \)

9) Find the Taylor series around 2 of \( \frac{1}{1-x} \) (5 Points)

- **Answer:** \( \sum_{n=0}^{\infty} (-1)^{n+1}(x-2)^n \)

10) Taylor Polynomials

10) What is the degree 1 Taylor polynomial of \( e^x \)? (1 Point)

- **Answer:** \( 1 + x \)

10) Find the degree 1 Taylor polynomial of \( \frac{1}{1-x} \) around 2 (2 Points)

- **Answer:** \( -1 + (x-2) \)

10) Find the degree 3 polynomial of \( (1+x)^{-1/3} \) (simplify coefficients) (3 Points)

- **Answer:** \( 1 - \frac{1}{3}x + \frac{4}{9}x^2 - \frac{28}{27}x^3 \)

10) What is the max error (a number) of the degree 5 Taylor polynomial around 0 of \( e^{3x} \) on the interval \([0, 1]\) (4 Points)

- **Answer:** \( \frac{3^6 e^3}{6!} \)

10) Find the max error (a number) of the degree 3 Taylor polynomial around 0 of \( x^{1/2} \) on the interval \([1, 3]\) (5 Points)

- **Answer:** \( \frac{15/16}{4!}(3^4) \)

11) Polar Equations

11) Convert the equation \( r = 1 + \cos \theta \) to \((x, y)\) coordinates (1 Point)

- **Answer:** \( x^2 + y^2 = \sqrt{x^2 + y^2} + x \)

11) Convert the equation \( y^2 = 2x - x^2 \) to \((r, \theta)\) coordinates (2 Points)
• Answer: \( r = 2 \cos(\theta) \)

11) Find the points on \( r = 2 + 2 \cos \theta \) which have a horizontal tangent line (3 Points)

• Answer: \( \theta = 2n\pi \)

11) Find the arc length of \( r = \cos^4(\frac{\theta}{4}) \) (4 Points)

• Answer: \( \frac{16}{3} \)

11) Find the area inside the curve \( r = \sin(4\theta) \) (5 Points)

• Answer: \( 2\pi \)