Let us see some definitions.

1. Define an alternative series and state the alternative series test.

2. State the root and ratio test.

3. What does it mean for a power series \( f(x) = \sum_{n=0}^{\infty} c_n x^n \) to have interval of convergence \([a, b]\).

4. State the difference between conditional convergence and absolute convergence. Then give an example of one conditionally convergent sequence and one absolutely convergent sequence.

5. Express the integral \( \int_0^1 \frac{\sin(x^2)}{x^2} \, dx \) as a series. Use that series to approximate it up to two decimal points.

6. State the binomial theorem.

Here is a question that combines everything we have done until now.

7. Let \( f(x) = 2\sin(3x) \)

   a) Find the MacLaurin series of \( f(x) \).

   b) Find the radius of convergence of this power series (which test do you use here?).

   c) Specify the interval of convergence (In case you use the alternative series test then specify when you use it and check the conditions).

   d) Find \( T_4(x) \), the degree 4 Taylor polynomial.

   e) Find the maximum error of the degree 4 approximation on the interval \([0, \frac{1}{3}]\).

   f) Write down a finite sum that approximates \( \sin(1) \) up to 0.01.

8. Find the limit \( \lim_{x \to \infty} \frac{x \sin(x^3) + \cos(x^2) - 1}{x^4} \) using power series.

9. Let \( f(x) = e^x \).

   a) Find the Taylor series of \( f(x) \) at the point \( a = 2 \).

   b) Find the radius of convergence of this Taylor series.

   c) State the degree 4 Taylor polynomial.

   d) What is the maximum error of the approximation on the interval \([0, 1]\)?

10. Express the integral \( \int_0^1 x^2 e^{-x^2} \, dx \) as a series. Use that series to approximate it up to one decimal point.

11. Find the limit \( \lim_{x \to \infty} \frac{(1 + x)^{1/3} - \sin(x) - \cos\left(\frac{x}{3}\right)}{3x^2} \).
12. Let \( f(x) = \frac{3}{8 - x^3} \).

a) Find the MacLaurin series of \( f(x) \).

b) Find the radius of convergence of this power series (which test do you use here?).

c) Specify the interval of convergence (In case you use the alternative series test then specify when you use it and check the conditions).

d) Find \( T_3(x) \), the degree 3 Taylor polynomial.

Some last problems before we wrap things up.

13. Use part (a) to solve part (b).

a) Find a power series representation of \( e^{x-2} \) centered at the point 2.

b) Find a power series representation of \( e^x \) centered at the point 8.

14. Find a power series representation for the following functions

a) \( \sin(x) - x \cos(x) \)

b) \( \ln(x) + 2 \tan^{-1}(x) \)

c) \( \sin(x) \cos(x) \)

d) \( \cos^2(x) \)

While you can always find a Taylor series for additions, in most circumstances you cannot find a Taylor series for multiplications. Instead you can find Taylor polynomials.

15. Find the degree 2 Taylor polynomial centered at 0 for the function \( f(x) = \sin(x) \ln(x + 1) \)

a) First by finding the derivatives of \( f(x) \).

b) By using the power series of \( \sin(x) \) and \( \ln(x + 1) \).

You did it. Enjoy your break!!!