Name: __________________

Fall 2015 Math 231 - Mock Final Exam

Instructions: You are encouraged to treat this exam like the real one - take it with a time limit and without calculators, notes, or solutions.

Multiple Choice Questions:

1. Evaluate \( \int_1^{e^2} \ln(x) \, dx \).
   (A) 1
   (B) \( e \)
   (C) \( 1 + 2e \)
   (D) \( 1 + e^2 \)
   (E) \( e^2 - 1 \)

2. Evaluate \( \int_0^1 xe^{2x} \, dx \).
   (A) \( 2e^2 - 2 \)
   (B) \( e^2 + 1 \)
   (C) \( e^2 + \frac{3}{2} \)
   (D) \( \frac{1}{4}e^2 + 1 \)
   (E) \( \frac{1}{4}(e^2 + 1) \)

3. Evaluate \( \int \frac{9x - 1}{x^2 - 1} \, dx \).
   (A) \( 4 \ln |x - 1| + 5 \ln |x + 1| + C \)
   (B) \( 5 \ln |x - 1| + 4 \ln |x + 1| + C \)
   (C) \( 6 \ln |x - 1| + 3 \ln |x + 1| + C \)
   (D) \( 4 \ln |x - 1| - 5 \ln |x + 1| + C \)
   (E) \( 6 \ln |x - 1| - 3 \ln |x + 1| + C \)
4. After making the correct trig substitution, the integral \( \int \frac{1}{x^2 \sqrt{9 + x^2}} \, dx \) becomes:

(A) \( \frac{1}{9} \int \frac{\cos^2(\theta)}{\sin(\theta)} \, d\theta \)

(B) \( \frac{1}{9} \int \frac{\cos(\theta)}{\sin^2(\theta)} \, d\theta \)

(C) \( \frac{1}{9} \int \cos^2(\theta) \sin^2(\theta) \, d\theta \)

(D) \( \frac{1}{9} \int \frac{\cos^3(\theta)}{\sin^2(\theta)} \, d\theta \)

(E) \( \frac{1}{9} \int \cos(\theta) \sin(\theta) \, d\theta \)

5. Find all values of \( p \) for which the integral \( \int_2^\infty \frac{1}{x \ln(x)^p} \, dx \) converges:

(A) \( p < 1 \)

(B) \( p \leq 1 \)

(C) \( p \geq 0 \)

(D) \( p > 1 \)

(E) \( p \geq 1 \)

6. Find the length of the curve \( y = 1 + \frac{2}{3}x^{3/2}, 0 \leq x \leq 1 \).

(A) \( \frac{2}{3}(2\sqrt{2} - 1) \)

(B) \( 2\sqrt{3} - \frac{4}{3}\sqrt{2} \)

(C) \( \frac{2}{3} \)

(D) \( \sqrt{3} - \frac{1}{3} \)

(E) \( \sqrt{3} + \frac{1}{3} \)
Determine whether the following improper integrals converge or diverge:

7. \[ \int_0^\infty \frac{x^2}{\sqrt{x^7 + 14}} \, dx \]
   (A) Converges
   (B) Diverges

8. \[ \int_1^\infty \frac{e^{-x} + x}{(x + 2)^2} \, dx \]
   (A) Converges
   (B) Diverges

9. \[ \int_0^1 \frac{1}{|x|^{1/3}} \, dx \]
   (A) Converges
   (B) Diverges

Determine whether the sequence \( \{a_n\} \) converges or diverges:

10. \( a_n = \frac{n^2 + n \sin(n)}{n^2 \ln(n) + n} \]
    (A) Converges
    (B) Diverges

11. \( a_n = \frac{(\ln(n))^{50}}{\sqrt{n}} \)
    (A) Converges
    (B) Diverges

12. \( a_n = \frac{n + (-1)^n n^2}{n^2 + 1} \)
    (A) Converges
    (B) Diverges
Determine whether the series converges absolutely, converges conditionally, or diverges:

13. \( \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!} \)
   (A) Converges Absolutely
   (B) Converges Conditionally
   (C) Diverges

14. \( \sum_{n=2}^{\infty} \frac{1}{n \ln(n)} \)
   (A) Converges Absolutely
   (B) Converges Conditionally
   (C) Diverges

15. \( \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3 + 7} \)
   (A) Converges Absolutely
   (B) Converges Conditionally
   (C) Diverges

16. \( \sum_{n=1}^{\infty} (-1)^n \cos(1/n) \)
   (A) Converges Absolutely
   (B) Converges Conditionally
   (C) Diverges

17. \( \sum_{n=2}^{\infty} n^5 \left( \frac{-2}{3} \right)^n \)
   (A) Converges Absolutely
   (B) Converges Conditionally
   (C) Diverges

18. \( \sum_{n=1}^{\infty} \frac{n^{7n}}{(2n + 1)^{6n}} \)
   (A) Converges Absolutely
   (B) Converges Conditionally
   (C) Diverges
19. Find the sum of the series \( \sum_{n=1}^{\infty} \frac{5^{2n}}{2^{5n+1}} \).
   (A) 25/64
   (B) 25/32
   (C) 25/14
   (D) 14/64
   (E) 14/32

20. Find the Maclaurin series for the function \( \frac{1 - \cos(x^2)}{x^2} \).
   (A) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-2}}{(2n)!} \)
   (B) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(4n)!} \)
   (C) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n)!} \)
   (D) \( \sum_{n=1}^{\infty} \frac{(-1)^n x^{4n}}{(4n)!} \)
   (E) \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{4n-2}}{(2n)!} \)

21. Find the equation for the polar curve \( r = 4 \sin(\theta) \) in rectangular coordinates.
   (A) \( x^2 + y^2 - 4 = 0 \)
   (B) \( x^2 + y^2 - 2x = 0 \)
   (C) \( x^2 + y^2 - 2y = 0 \)
   (D) \( x^2 + y^2 - 4x = 0 \)
   (E) \( x^2 + y^2 - 4y = 0 \)

22. Find the total area enclosed by the polar curve \( r = \sin(2\theta) \).
   (A) \( \pi \)
   (B) \( \pi/2 \)
   (C) \( \pi + 1 \)
   (D) \( \pi/2 + 1 \)
   (E) 1
23. Use Simpson’s Rule with \( n = 4 \) to estimate the value of \( \int_{0}^{4} x^2 \, dx \).

(A) 52
(B) 52/3
(C) 64
(D) 64/3
(E) 112/3

24. Find the value of the series \( \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{2^{2n}(2n+1)!} \).

(A) 2
(B) \( \pi \)
(C) \( \pi/2 \)
(D) \( \cos(2) \)

25. Match each of the following integrals with an appropriate substitution that could be used to evaluate it.

(I) \( \int \sin^3(x) \cos^6(x) \, dx \)

(A) \( u = \cos(x) \)

(B) (use half-angle formulas)

(C) \( u = \sin(x) \)

(D) \( u = \tan^2(x) \)

(E) \( u = \tan(x) \)

(F) \( u = \sec(x) \)
Free Response:

26. Consider the lamina $L$ in the plane of constant density $\lambda$, which is formed from the region inside the circle $x^2 + y^2 = 4$ and between the lines $y = 0$ and $y = 1$. Find $M_x$, the moment of $L$ about the $x$-axis.
27. Consider the region $R$ defined by the curves $y = 4x^2$ and $y = 4$. Suppose a plate in the shape of $R$ is submerged underwater, where the top of the plate is the line $y = 4$ and is 3 meters underwater. Set up, but do not evaluate an integral to compute the hydrostatic force on the plate. You may use $\rho$ for the density of the water and $g$ for the acceleration due to gravity.
28. Consider the curve defined by the parametric equations $x = \cos^3(t), y = \sin^3(t)$, $0 \leq t \leq \pi$.

a) Set up but \textbf{do not evaluate} an integral which represents the area between the curve and the $x$-axis.

b) Find the length of the curve by evaluating an integral.
29. a) Give the Maclaurin series for the function \( \cos(x) - 1 \).

b) Use series to evaluate the limit \( \lim_{x \to 0} \frac{\cos(x) - 1}{x^2 e^x} \).

c) Estimate \( \int_0^1 (\cos(x) - 1) \, dx \) to within .001. You do not need to simplify your answer, but you must justify the accuracy.
30. a) Find the Taylor series for \( f(x) = xe^x \) centered at 0.

b) Find the degree 2 Taylor polynomial \( T_2(x) \) for \( f(x) = xe^x \) centered at 0.

c) \( T_2(x) \) is used to approximate \( f(x) \) in the range \( 0 \leq x \leq 1/2 \). Find the maximum possible error in this approximation (you do not need to simplify your answer).
31. a) First make a careful sketch of the polar curve \( r = 1 + \sqrt{2} \cos(\theta) \) on rectangular axes, then make a careful sketch of the curve on polar axes. Include all important features of your sketches, including all important angles.

b) Find the slope of the tangent line to the polar curve at the point where \( \theta = \pi/2 \).

c) Set up and simplify \textbf{but do not evaluate} an integral which represents the total length of the polar curve.
32. Consider the power series \( \sum_{n=0}^{\infty} \frac{(x + 1)^n}{n + 1} \).

a) Find the radius of convergence.

b) Find the interval of convergence.