1. Define the following terms

   (a) Local Maximum (minimum)

   (b) Absolute Maximum (minimum)

   (c) Critical Value

2. State the following theorems

   (a) Fermat’s Theorem.

   (b) Extreme Value Theorem.

   (c) Mean Value Theorem.

   (d) Absolute Maximum (Minimum) Theorem.

   (e) Rolle’s Theorem.
3. (Board Problem) Let \( f(x) = x^2 \) give a graphical representation for the mean value theorem on the interval \([0, 4]\).

4. True or False?

   (a) Let \( f(x) \) be a continuous function and \( f \) have a maximum at \( c \). Then \( f'(c) = 0 \).

   (b) Let \( f(x) \) be a differentiable function and \( f'(c) = 0 \). Then \( f \) has a minimum at the point \( c \).

   (c) Let \( f(x) \) be a differentiable function and \( f(a) = f(b) \). Then there exists a point \( c \) such that \( f'(c) = 0 \).

   (d) If \( f(x) \) is a continuous function on the interval \([a, b]\), then \( f \) has a minimum on this interval.

   (e) There is a relation between Rolle’s theorem and the mean value theorem.

5. (Board Problem) Sketch the graph of a function \( f \) that is continuous on \([1, 5]\) and has the given properties: \( f \) has an absolute minimum at 1, an absolute maximum at 5, a local maximum at 2, and a local minimum at 4.
6. Find the critical numbers of the function \( f(x) = x^2 + 5x \)

7. Find the critical numbers of the function \( f(x) = 2x^3 + x^2 + 2x \).

8. Find the critical numbers of the function \( f(x) = x^3 + 6x^2 + 15x \).

9. Find the critical numbers of the function \( f(x) = \sin(x) \)
10. Suppose that $f$ and $g$ are continuous on $[a, b]$ and differentiable on $(a, b)$. Suppose also that $f(a) = g(a)$ and $f'(x) < g'(x)$ for $a < x < b$. Prove that $f(b) < g(b)$. Hint: apply the Mean Value Theorem to the function $h = f - g$.

11. Let $A$ be the area of a rectangle with sides length $x - 2$ and $x - 5$. What is the maximum area?

12. Combine IVT and Rolle’s Theorem to show that $x^7 + x^5 + x^3 + x = 0$ has only one root.

13. Do the same for $x^4 + 3x + 1$ to show that it has exactly one solution in the interval $[-2, 1]$.

14. Let $f(x)$ be a differentiable function on $(a, b)$ such that $f'(x) \neq 0$ on the interval. Show that $f(a) \neq f(b)$. 