1. Find the following limits:
   \[ \lim_{x \to 0} \frac{e^{5x} - 1 - 5x - 12.5x^2}{x^3} \]
   \[ \lim_{x \to 0} \frac{3}{1 + \frac{4}{x}} \]

2. Find \( \frac{dy}{dx} \) for the following functions
   (a) \( y = \ln(x + e^x \sin(x)) \)
   (b) \( x^2 + xy + xy^3 + x^2y^2 = 1 \)

3. Let \( f(x) = \frac{x^2 - 2x}{x^2 + 1} \)
   (a) Find all horizontal and vertical asymptotes
   (b) Find the first derivative and all critical points
   (c) Use that to find all maximum and minimum points and intervals of increase/decrease
   (d) Find the second derivative and all its critical points
   (e) Use that to find all inflection points and all intervals of concavity up/down
   (f) Use the information gathered to give a rough sketch of the function

4. Prove the following using the right theorems:
   (a) We never study with a constant efficiency. Show, that there is a moment though where your efficiency is at its peak, let’s say you are studying on Sunday from 9 am to 6 pm (try to get the most out of that moment).
   (b) You cared about Math 221 throughout this semester at different levels. For example, you don’t care at all before the semester started or after it will end, but you care quite a lot before midterms, which shows that it is not constant at all. Now, show that there is at least one moment where you cared most about it.
   (c) Imagine yourself right before the exam. Clearly, this is the moment you know most about Math 221. It seems that at that moment your knowledge of it is neither increasing nor decreasing. Can you prove it ?
   (d) Gauss was one of the biggest mathematicians ever. Prove that there was a time in his life where his knowledge of Calculus was as much as yours.

5. A particle is moving on the trajectory \( y^2x^2 + yx^2 + y^3x + x^2 = 4 \). When it is at the point \((1, 1)\) it has a horizontal velocity of \(2 \text{ m/s} \). What is the vertical velocity?

6. Tom wants to prepare for his Math 221 exam. If he studies for one hour, he is able to study 20 pages of Stewart in an effective manner. For every one hour he adds to his studies his efficiency decreases by 1 page an hour. How many hours per day should he study to get the most out of it?
7. Find the following integrals.

(a) \[ \int_{5}^{10} (2y - 3)(y - 5)^{20} \, dy \]

(b) \[ \int_{0}^{\sqrt{5}} \sqrt{5 - x^2} \, dx \]

(c) \[ \int \frac{4 \cot(t)}{(\ln(\sin(t)))} \, dt \]

8. Let \( D \) be the area between the graphs \( y = x^2, y = 6 - x \) and the \( y \)-axis. Find but DO NOT EVALUATE an integral which represents the following volumes:

(a) Rotating the area around the \( x \) axis.

(b) Rotating the area around \( y = -1 \).

(c) If the cross-section perpendicular to the \( x \)-axis is a equilateral triangle.

(d) Rotating around \( x = 7 \).

(e) If the cross-section perpendicular to the \( y \)-axis is a semicircle.