Quiz #3 Answers, Friday September 18
Math 221 Lecture EL1

Instructions. Be sure to show your work and explain your reasoning where necessary for full credit.

Problem 1 (2 points total)
If a rock is thrown upward on the planet Mars with a velocity of 10 meters per second, its height (in other words, its position) in meters after $t$ seconds is given by the equation

$$H(t) = 10t - 1.86t^2.$$ 

Find the velocity of the rock after 1 second.

Answer: 6.28 meters/second.

Solution: To find the velocity of a rock at $t = 1$, you can either (1) employ the strategy used in Problem 13 from Section 2.7, which I recommended in Lecture 9/8 to prepare for Discussion and Webassign, or (2) use your knowledge that the velocity function is the derivative of the position function and use the Power Rule from Section 3.1 to find $v(t)$. I will present both solutions, although in general I would recommend using Solution (2) unless the problem directions expressly require you to use the limit definition of a derivative for the computation.

Solution (1):

$$v(1) = \lim_{h \to 0} \frac{H(1 + h) - H(1)}{h} = \lim_{h \to 0} \frac{10(1 + h) - 1.86(1 + h)^2 - 8.14}{h} =$$

$$= \lim_{h \to 0} \frac{10 + 10h - 1.86(1 + 2h + h^2) - 8.14}{h} = \lim_{h \to 0} \frac{10 + 10h - 1.86 - 3.72h - 1.86h^2 - 8.14}{h} =$$

$$= \lim_{h \to 0} \frac{6.28h - 1.86h^2}{h} = \lim_{h \to 0} (6.28 - 1.86h) = 6.28$$

Solution (2):

$$v(t) = H'(t) = 10 - 1.86 \times 2 \times t = 10 - 3.72t$$

So, $v(1) = 10 - 3.72(1) = 6.28$

Problem 2 (6 points total- 3 points each item)
Find $h'(x)$ for the given function $h(x)$:

(a) $h(x) = \frac{x}{3 + \sqrt{x}}$
Answer: \( h'(x) = \frac{3 + \frac{1}{2}\sqrt{x}}{(3 + \sqrt{x})^2} \)

Solution: We must use the Quotient Rule for this problem. Writing \( h(x) = \frac{f(x)}{g(x)} \) with \( f(x) = x \) and \( g(x) = 3 + \sqrt{x} \), we have

\[
h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} = \frac{(3 + \sqrt{x})(1) - x(0 + \frac{1}{2\sqrt{x}})}{(3 + \sqrt{x})^2} = \frac{3 + \sqrt{x} - \frac{1}{2}\sqrt{x}}{(3 + \sqrt{x})^2} = \frac{3 + \frac{1}{2}\sqrt{x}}{(3 + \sqrt{x})^2}.
\]

(b) \( h(x) = x^2e^x \)

Answer: \( h'(x) = x^2e^x + 2xe^x \)

Solution: We must use the Product Rule for this problem. Writing \( h(x) = f(x)g(x) \) with \( f(x) = x^2 \) and \( g(x) = e^x \), we have

\[
h'(x) = f(x)g'(x) + g(x)f'(x) = x^2(e^x) + e^x(2x) = x^2e^x + 2xe^x.
\]

Problem 3 (2 points total)

(Multiple Choice) For each function \( f(x) \) in the two parts of Problem 3 choose the graph (a) or (b) that represents \( f'(x) \). You don’t need to justify your answer or show work-this is a multiple choice problem so no partial credit will be given.

Problem 3 Part 1:
Figure 1: A Graph of $f(x)$ for Problem 3(a)

(a) Choice (a) for Problem 3 Part 1
(b) Choice (b) for Problem 3 Part 2

Answer for Part 1: Choice (b). (You have seen this $f(x)$ before: it is the greatest integer function you met on Worksheet 6). As in all multiple choice problems, rule out as many choices as you can before deciding the right answer. Here there are only two, so you need to rule out one definitively and then you’re done-circle or state your answer.

Explanation 1: Let’s look at choice (a): The graph of $f(x)$ has a discontinuity at $x = 1$, where as the graph in choice (a) does not-in fact the graph shows the derivative is defined there. So this Rules On page 159-I sent an email to all of you on 9/10 telling you to look at page 159 to prepare for Worksheet 6- you’ll see that a function cannot be differentiable at a discontinuity.

Explanation 2: Again, look at choice (a): We can see that between discontinuities for $f(x)$-meaning the points on the graph of $f(x)$ corresponding to non-integer values- the tangent line should be flat, so the derivative of $f(x)$ at, say, $x = \frac{1}{2}$ should be zero. but in choice we can see that choosing (a) would mean that $f'(\frac{1}{2}) \approx \frac{1}{2} \neq 0$. So this rules out choice (a).

Problem 3 Part 2:
Figure 3: A Graph of $f(x)$ for Problem 3 Part 2

Answer for Part 2: Choice (a). As in all multiple choice problems, rule out as many choices as you can before deciding the right answer. Here there are only two, so you need to rule out one definitively and then you’re done-circle or state your answer.

Explanation: Let’s look at choice (b): The graph of $f(x)$ shows that all tangent lines to the graph of $f(x)$ should have negative slope -equivalently, the numerical value of $f'(x)$ should be negative between about 1.2 and the end of our picture. But the graph of choice (b) shows a graph of a function with all positive values everywhere, so this can’t be the graph of $f'(x)$. 