Quiz #11 Solutions, Friday November 13
Math 221 Lecture EL1

Instructions. Be sure to show your work and explain your reasoning where necessary for full credit. This quiz has three problems, and two pages

1. Find the indefinite integral

\[ \int \sec t (\sec t + \tan t) \, dt \]

Solution:

We did this problem in Lecture Tuesday, 11/10. First, we multiply through inside the integral

\[ \int \sec t (\sec t + \tan t) \, dt = \int \sec^2 t + \sec t \tan t \, dt \]

And then we recognize the two summands inside the integral as basic indefinite integrals from the Table on page 398, obtaining the answer

\[ \tan t + \sec t + C. \]

2. 

\[ \int_0^1 x (\sqrt[3]{x} + \sqrt[4]{x}) \, dx \]

Solution (This was a practice problem I assigned Tuesday that had a video solution in the e-book), please refer to that solution.

3. Find \(dy/dx\)

\[ y = \int_{1-3x}^{1} \frac{t^3}{1 + t^2} \, dt \]
Solution: To use the FTC 1 we want the function or variable to be the upper bound of integration, so first, remember that we can write (see page 379)

\[
\int_{1-3x}^{1} \frac{t^3}{1+t^2} \, dt = -\int_{1}^{1-3x} \frac{t^3}{1+t^2} \, dt.
\]

Then, let \( u = 1 - 3x \), so that

\[
\frac{du}{dx} = -3 \quad \text{and} \quad \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}
\]

Therefore

\[
\frac{dy}{dx} = \frac{d}{du} \int_{u}^{1} \frac{t^3}{1+t^2} \, dt \left( \frac{du}{dx} \right) = -\frac{d}{du} \int_{1}^{u} \frac{t^3}{1+t^2} \, dt \left( \frac{du}{dx} \right) = -\frac{u^3}{1+u^2}(-3) = \frac{3(1 - 3x)}{1 + (1 - 3x)^2}
\]