Quiz #7 for Math 231 Solutions

Instructions. Be sure to show your work and explain your reasoning for full credit.

NAME ______________________

1. Use the integral test to determine if the series converges or diverges.

\[ \sum_{n=1}^{\infty} \frac{n}{n^4 + 1} \]

Solution:
Using the substitution \( u = x^2 \) we have

\[ \int_1^{\infty} \frac{x}{x^4 + 1} \, dx = \frac{1}{2} \int_1^{\infty} \frac{du}{u^2 + 1} = \frac{1}{2} \tan^{-1}(u) \bigg|_1^{\infty} = \frac{1}{2} \left( \frac{\pi}{2} - \frac{\pi}{4} \right) = \frac{\pi}{8} \]

Since the improper integral converges, the series converges also by the Integral test.

2. Determine whether the series is convergent or divergent.

(a) \[ \sum_{n=0}^{\infty} \frac{1}{\sqrt{n}^3 + 1} \]

Solution:

\[ \frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}^3} = \frac{1}{n^{3/2}} \]

\[ \sum \frac{1}{n^{3/2}} \] converges by the p-test. Therefore the original series also converges.

(b) \[ \sum_{n=1}^{\infty} \frac{3^n}{8^n - 5} \]

Solution:

We know that \( \sum \left( \frac{3}{8} \right)^n \) converges, geometric with \( r = \frac{3}{8} < 1 \).

Limit comparison test: \( a_n = \frac{3^n}{8^n} = \left( \frac{3}{8} \right)^n \), \( b_n = \frac{3^n}{8^n - 5} \)

\[ \lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{\frac{3^n}{8^n}}{\frac{3^n}{8^n - 5}} = \lim_{n \to \infty} \frac{8^n - 5}{8^n} = \lim_{n \to \infty} \frac{1 - \frac{5}{8^n}}{1} = 1 \]

Since \( \sum \left( \frac{3}{8} \right)^n \) converges, both series have to converge. So the original series converges.