Math 231 Exam III
UIUC, April 18, 2013

1. (8 points) Short answer.

(a) Suppose that \( c(x) = \sum_{n=0}^{\infty} c_n x^n \) converges for \( x = -4 \) but diverges for \( x = 6 \).

(i) \( \sum_{n=0}^{\infty} 2^n c_n \) (absolutely converges/ conditionally converges/diverges).

(ii) \( \sum_{n=0}^{\infty} (-8)^n c_n \) (absolutely converges/ conditionally converges/diverges).

(b) Calculate the binomial coefficient \( \binom{-3}{3} = \).

(c) Recall that \( \ln 2 = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} \). By the Alternating Series Estimation, how accurate is \( 1 - \frac{1}{2} + \cdots - \frac{1}{8} + \frac{1}{9} = \frac{1879}{2520} \) to the actual value of \( \ln 2 \)?

2. (10 points) Find a series solution to the integral

\[
\int_0^1 e^{-x^2} \, dx
\]

3. (12 points each) Find the radius and interval of convergence for the power series. Be sure to indicate which points converge absolutely and which converge conditionally.

(a) \( \sum_{n=0}^{\infty} \frac{(x + 2)^n}{2 \cdot 4 \cdot 6 \cdots (2n + 2)} \)

(b) \( \sum_{n=0}^{\infty} \frac{(3x + 2)^n}{(n + 1)(n)} \)
4. (12 points) Give the Taylor polynomial with degree 2 centered at 1 for $f(x) = \sqrt[3]{x}$. Then use Taylor’s Inequality to estimate the accuracy of this approximation for $x$ between 1 and 1.1.

5. (12 points) Let $f$ be a function which has all derivatives and has the property that $f'' = f$. If $f(0) = 0$ and $f'(0) = 1$, what is the power series for $f$ at 0?

6. (12 points) Determine a power series centered at 0 for $f(x) = \sin^{-1} x$ and use it to determine the 100-th derivative of $\sin^{-1} x$ at 0. You may find the following useful

$$\left(\frac{-1}{2^n}\right) = (-1)^n \frac{(1)(3)(5) \cdots (2n-1)}{2^n n!}$$