Quiz #9
Math 221

Instructions. Be sure to show your work and explain your reasoning for full credit.

NAME ________________________

1. A farmer is going to use fence to make a pen shaped like cutting a circle in half and inserting a rectangle.

If the pen is to have area $100\pi$, how should he make the pen to minimize the amount of fence needed? Be sure to show that your solution is minimal.

Let $x$ be the length of a straight side and $r$ the radius of the circles. The perimeter will be $P = 2x + 2\pi r$ and the area $A = 2rx + \pi r^2$. Since $A = 100\pi$, $x = \frac{100\pi - \pi r^2}{2r}$ and we want to minimize

$$P(r) = 2\left(\frac{100\pi - \pi r^2}{2r}\right) + 2\pi r = \pi \left(\frac{100}{r} + r\right)$$

for $0 < r \leq 10$.

$$P'(r) = \pi \left(-\frac{100}{r^2} + 1\right)$$

which is 0 at $r = 10$ and since $P' < 0$ over $(0, 10]$ $P(10)$ is the minimum. Thus, the farmer should make the pen a circle of radius 10 and no straight sides.

2. Indicate on the graph where the first two iterations of Newton’s method will lie if one starts with the indicated value $a$. Then explain why $a$ would or would not be a good starting point to approximate the root of the function.
The point $a$ would NOT be a good point to start as the iterations of Newton’s method will move away from the root we are trying to estimate.