Some questions on this worksheet reference a computer-generated demonstration (access it through Moodle) to aid in conceptual understanding of the questions.

1. In this problem, we’ll compute the volume of the solid known as Gabriel’s Horn.

(a) Determine the volume of the object formed by rotating the curve \( y = \frac{1}{x} \) about the \( x \)-axis between \( x = 1 \) and \( x = 5 \).

Solution:
\[
\pi \int_1^5 \frac{1}{x^2} \, dx = -\pi \left[ \frac{1}{x} \right]_1^5 = -\frac{\pi}{5} + \pi = \frac{4\pi}{5}
\]

(b) More generally, what is the volume of the object formed by rotating the curve \( y = \frac{1}{x} \) about the \( x \)-axis between \( x = 1 \) and \( x = t \) (where \( t > 1 \))?

Solution:
\[
\pi \int_1^t \frac{1}{x^2} \, dx = -\pi \left[ \frac{1}{x} \right]_1^t = -\frac{\pi}{t} + \pi.
\]

(c) Gabriel’s Horn is formed by rotating the curve \( y = \frac{1}{x} \) about the \( x \)-axis for \( x \geq 1 \). Determine the volume of this object by letting \( t \to \infty \) from the previous part.

Solution:
\[
\pi \int_1^\infty \frac{1}{x^2} \, dx = \lim_{t \to \infty} \pi \int_1^t \frac{1}{x^2} \, dx = \lim_{t \to \infty} \left( -\frac{\pi}{t} + \pi \right) = \pi.
\]

2. Let \( R \) represent the region bounded by the curves \( y = \sqrt{9 - x^2} \) and \( y = 1 \).

(a) Set up (but do not evaluate) the integral that computes the area of \( R \).

Solution: For the points of intersection:
\[
\sqrt{9 - x^2} = 1 \Rightarrow x^2 = 8 \Rightarrow x = \pm 2\sqrt{2}.
\]

Therefore, the required area is
\[
\int_{-2\sqrt{2}}^{2\sqrt{2}} (\sqrt{9 - x^2} - 1) \, dx
\]

(b) Set up (but do not evaluate) the integral that computes the volume of the object formed by rotating \( R \) about the \( x \)-axis.

Solution:
\[
\pi \int_{-2\sqrt{2}}^{2\sqrt{2}} ((\sqrt{9 - x^2})^2 - 1) \, dx = \int_{-2\sqrt{2}}^{2\sqrt{2}} (8 - x^2) \, dx.
\]
(c) Set up (but do not evaluate) the integral that computes the volume of the object formed by rotating $\mathcal{R}$ about the line $y = -2$.

**Solution:**
\[
\pi \int_{-2\sqrt{2}}^{2\sqrt{2}} \left( (2 + \sqrt{9 - x^2})^2 - (2 + 1)^2 \right) dx.
\]

3. Let $\mathcal{R}$ be the region bounded by the curves $y = x^2$ and $y = x$.

(a) Set up (but do not evaluate) the integral that computes the volume of the object formed by rotating $\mathcal{R}$ about the line $x = -1$.

**Solution:** Since this is rotated about a horizontal line, we wish to integrate with respect to $y$ for $x = \sqrt{y}$ and $x = y$. The points of intersections are at $y = 0$ and $1$ and so
\[
\pi \int_0^1 \left( (\sqrt{y} + 1)^2 - (y + 1)^2 \right) dy
\]

(b) Set up (but do not evaluate) the integral that computes the volume of the object formed by rotating $\mathcal{R}$ about the line $y = 3$.

**Solution:**
\[
\pi \int_0^1 \left( (3 - x^2)^2 - (3 - x)^2 \right) dx
\]

4. Let $\mathcal{R}$ be the region bounded by the curves $x = y^2 - 4$ and the $x = 8 - y$.

(a) Set up (but do not evaluate) the integral that computes the area of $\mathcal{R}$.

**Solution:** For the points of intersection:
\[
y^2 - 4 = 8 - y \Rightarrow y^2 + y - 12 = 0 \Rightarrow (y + 4)(y - 3) = 0 \Rightarrow y = -4, 3.
\]

Therefore, the required area is
\[
\int_{-4}^{3} \left( (8 - y) - (y^2 - 4) \right) dy
\]

(b) Set up (but do not evaluate) the integral that computes the volume of the object formed by rotating $\mathcal{R}$ about the line $x = -8$.

**Solution:**
\[
\pi \int_{-4}^{3} \left( (8 - (8 - y))^2 - (8 - (y^2 - 4))^2 \right) dy
\]