Fix a Polish space $X$ and $\mu \in \mathcal{P}(X)$.

1. [10 points] Show that $\mu$ is regular; i.e., that $\mu = \mu_* = \mu^*$, where

$$\mu_*(A) \overset{\text{def}}{=} \sup\{\mu(F) : F \subset A \text{ and } F \text{ closed}\}$$
$$\mu^*(A) \overset{\text{def}}{=} \inf\{\mu(G) : G \supset A \text{ and } G \text{ open}\}$$

for all $A \in \mathcal{B}(X)$. Hint: the collection of sets $A$ for which $\mu(A) = \mu^*(A) = \mu_*(A)$ is a

2. [10 points] Show that $\mu$ is itself tight. Hint: the fact that $X$ is separable means . . . .

3. [10 points] Fix $\nu \in \mathcal{P}(X)$ such that $H(\nu|\mu) < \infty$. Suppose that $\{A_n ; n \in \mathbb{N}\} \subset \mathcal{B}(X)$

is such that $\lim_{n \to \infty} \mu(A_n) = 0$. Show that $\lim_{n \to \infty} \nu(A_n) = 0$. 