1. **10 points** Exercise 1.1.1

2. **10 points** Exercise 1.1.2

3. **45 points** Exercise 1.1.3.
   (a) **15 points** Show that $X_n$ is a Markov chain and compute its transition matrix $P$.
   (b) **15 points** Consider any distribution $\lambda$ and any transition matrix $P$. Show that it can be constructed via the inductive formula $X_{n+1} = G(X_n, Y_{n+1})$.
   (c) **15 points** Do parts a), b), and c). Note that the state space may be $\mathbb{Z}$.

4. **15 points** Consider a two-state Markov chain $(X_n)_{n=0}^\infty$ with transition matrix

$$P = \begin{pmatrix}
1 - \alpha & \alpha \\
\beta & 1 - \beta
\end{pmatrix}$$

Consider the stochastic process $Y_n = (X_{n+1}, X_n)$. Show that $Y_n$ is a Markov process and find its transition matrix.