Math 461C13 (Panki Kim)
Sept. 30, 2005
Midterm 1 (50 points)

Name ______________

UID Number ______________

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Instructions:

- Your exam contains 4 problems. The entire exam is worth 50 points. The point value of each problem is clearly marked.
- Your exam should contain 5 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- You have 50 minutes for this midterm. Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. No calculators and notes are allowed. If in doubt, ask for clarification.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Make sure to do your own work on the exam. Good Luck!

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Problem #1 (10 pts) ______________

Problem #2 (10 pts) ______________

Problem #3 (15 pts) ______________

Problem #4 (15 pts) ______________

TOTAL (50 pts) ______________

1. (10pts) Suppose that we toss $5$ independent fair coins. Find the probability having at least $2$ heads.

2. (10pts) A student is taking a multiple choice exam in which question has $4$ possible answers, exactly one of which is correct. If the student knows the answer he selects the correct answer. Otherwise he selects one answer at random from $4$ possible answers. Suppose that the student knows the answer to $3/4$ of the question. If the student gets the correct answer to a question, what is the probability that he does not know the answer?

3. (15pts) The distribution function of $X$ is given by

$$F_X(x) \;=\; \begin{cases} 0 & \text{if} \quad x < 0 \\ x^2/4 & \text{if } 0 \le x < 1 \\ 1/3 & \text{if } 1 \le x < 2 \\ x/3 & \text{if } 2 \le x < 3 \\ 1 & \text{if} \quad x \ge 3 \end{cases} .$$

(a) (5pts) Find $\mathbf{P}(\ 0 \le X \le 1\ )$.

(b) (5pts) Find $\mathbf{P}(\ 1 < X < \frac{5}{2}\ )$.

(c) (5pts) Find $\mathbf{P}(\ X = 1\ )$.

4. (10pts) Urn has two red balls and three blue balls. You pick two balls randomly without replacement. Let $X$ be the number of blue balls selected.

   (a) (8pts) Find $\mathbf{E}[X]$.

(b) (7pts) Now you add $X$ number of red ball to the urn (which has remaining three balls) and pick two balls from that urn randomly without replacement. What is the probability that you get two red balls in second trial?

# Math 461C1 Midterm 1 Solution

1. (10pts) $1 - \mathbf{P}(\text{at most one head}) = 1 - \mathbf{P}(\text{one head}) - \mathbf{P}(\text{No head}) = 1 - \binom{5}{1}(\frac{1}{2})^5 - (\frac{1}{2})^5 = \frac{13}{16}$
2. (10pts) Let $K := \{$ Know the answer $\}$ and $C := \{$ Correct answer $\}$. Then

$$\mathbf{P}(K^c|C) = \frac{\mathbf{P}(K^c \cap C)}{\mathbf{P}(C)} = \frac{\mathbf{P}(K^c)\mathbf{P}(C|K^c)}{\mathbf{P}(K^c)\mathbf{P}(C|K^c) + \mathbf{P}(K)\mathbf{P}(C|K)} = \frac{\frac{1}{4}\cdot\frac{1}{4}}{\frac{1}{4}\cdot\frac{1}{4} + 1\cdot\frac{3}{4}} = \frac{1}{13}.$$

3. (15pts)
   (a) (5pts)$\mathbf{P}(\ 0 \le X \le 1\ ) = \mathbf{P}(X \le 1) - \mathbf{P}(X < 0) = F(1) - F(0-) = \frac{1}{3} - 0 = \frac{1}{3}$.
   (b) (5pts) $\mathbf{P}(\ 1 < X < \frac{5}{2}\ ) = \mathbf{P}(X < \frac{5}{2}) - \mathbf{P}(X \le 1) = F(\frac{5}{2}-) - F(1) = \frac{5}{6} - \frac{1}{3} = \frac{1}{2}$.
   (c) (5pts) $\mathbf{P}(\ X = 1\ ) = \mathbf{P}(X \le 1) - \mathbf{P}(X < 1) = F(1) - F(1-) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$.
4. (15pts)
   (a) (8pts)

$$\begin{aligned} p_X(0) &= \mathbf{P}(X=0) = \frac{\binom{2}{2}}{\binom{5}{2}} = \frac{1}{10}. \\ p_X(1) &= \mathbf{P}(X=1) = \frac{\binom{3}{1}\binom{2}{1}}{\binom{5}{2}} = \frac{6}{10}. \\ p_X(2) &= \mathbf{P}(X=2) = \frac{\binom{3}{2}}{\binom{5}{2}} = \frac{3}{10}. \end{aligned}$$

   So $\mathbf{E}[X] = \mathbf{P}(X=1) + 2\mathbf{P}(X=2) = \frac{6}{10} + 2\cdot\frac{3}{10} = \frac{6}{5}$.
   (b) (7pts) Let $E := \{$Two red balls in secon trial$\}$.

$$\begin{aligned} \mathbf{P}(E) &= \mathbf{P}(E \cap \{X=0\}) + \mathbf{P}(E \cap \{X=1\}) + \mathbf{P}(E \cap \{X=2\}) \\ &= \mathbf{P}(X=0)\mathbf{P}(E|\{X=0\}) + \mathbf{P}(X=1)\mathbf{P}(E|\{X=1\}) + \mathbf{P}(X=2)\mathbf{P}(E|\{X=2\}) \\ &= p_X(0)\mathbf{P}(E|\{X=0\}) + p_X(1)\mathbf{P}(E|\{X=1\}) + p_X(2)\mathbf{P}(E|\{X=2\}) \\ &= \frac{1}{10}\cdot 0 + \frac{6}{10}\frac{\binom{2}{2}}{\binom{4}{2}} + \frac{3}{10}\frac{\binom{4}{2}}{\binom{5}{2}} \\ &= \frac{6}{10}\cdot\frac{1}{6} + \frac{3}{10}\cdot\frac{3}{5} = \frac{7}{25}. \end{aligned}$$

Math 461C13 (Panki Kim)
Nov. 4, 2005
Midterm 2 (40 points)

Name ______________

University ID number ______________

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Instructions:

- Your exam contains 4 problems. The entire exam is worth 40 points. The point value of each problem is clearly marked.
- Your exam should contain 5 pages; please make sure you have a complete exam.
- This exam is closed book.
- Box in your final answer when appropriate.
- You have 50 minutes for this midterm. Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. No calculators are allowed. If in doubt, ask for clarification.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Make sure to do your own work on the exam. Good Luck!

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Problem #1 (10 pts) ______

Problem #2 (10 pts) ______

Problem #3 (10 pts) ______

Problem #4 (10 pts) ______

TOTAL (40 pts) ______

1. (10pts) The time (hour) required to fix a computer is distributed with the density function:

$$f(x) = \begin{cases} c\,e^{-2x} & \text{if } 1 \le x < \infty \\ 0 & \text{otherwise} \end{cases} .$$

   (a) (5pts) Find $c$.

   (b) (5pts) Find the probability that the repair time is between $3$ and $5$ hours, given condition that the repair time is at least $2$ hours.

2. (10pts) Suppose a box has three blue balls and two red balls. Tom chooses two balls and check their colors and put those back. He repeats until two red balls occur for the first time. Let $X$ be the number of trials until two red balls occur.

   (a) (5pts) What is the probability that $X > 2$ ?

   (b) (5pts) Find $\mathbf{E}[X^2]$.

3. (10pts) $U$ is uniformly distributed on $(-1, 1)$.

(a) (5pts) Find $\mathbf{E}[e^U]$.

(b) (5pts) Find the distribution function $F_X$ of $X$ when $X = U^4$.

4. (10pts) The joint density function of $(X, Y)$ is given by

$$f_{X,Y}(x, y) = \begin{cases} c\,e^{y}\sin x & \text{if } 0 < x < \pi,\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases} .$$

(a) (5pts) Find $c$.

(b) (5pts) Find the marginal density function $f_X$ of $X$.

# Math 461 Midterm 2 Solution

Note Title 11/4/2005

1 (a) Need $\int_1^\infty c e^{-2x}\,dx = 1$

$$\int_1^\infty e^{-2x}\,dx = -\frac{1}{2} e^{-2x}\Big|_1^\infty = +\frac{1}{2}(e^{-2})$$

$$\therefore c = 2e^2$$

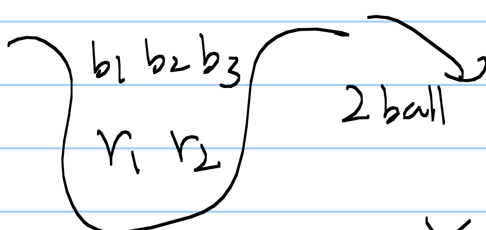
(b) $\mathbb{P}(3<X<5 \mid X>2) = \dfrac{\mathbb{P}(3<X<5)}{\mathbb{P}(X>2)}$

$$c\int_3^5 e^{-2x}\,dx \Big/ c\int_2^\infty e^{-2x}\,dx$$

$$= \left(-\frac{1}{2}\right)(e^{-10}-e^{-6}) \Big/ \left(-\frac{1}{2}\right)(-e^{-4})$$

$$= (e^{-10}-e^{-6})(-e^{+4}) = e^{-2}-e^{-6}$$

2 (a)




$$\binom{5}{2} = \frac{5\cdot 4}{2\,1} = 10$$

$p = \frac{1}{10}$

$X \sim$ Geometric with $p = \frac{1}{10}$

$$\mathbb{P}(X>2) = 1 - \mathbb{P}(X=1) - \mathbb{P}(X=2)$$

$$= 1 - \frac{1}{10} - \frac{9}{10}\cdot\frac{1}{10} = \boxed{\frac{81}{100}}$$

(b) $Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

So $\mathbb{E}[X^2] = Var(X) + (\mathbb{E}[X])^2 = \dfrac{1-p}{p^2} + \dfrac{1}{p^2} = \dfrac{2-p}{p^2}$

$$= \left(2-\frac{1}{10}\right)100 = 200 - 10 = \boxed{190}$$

or $\mathbb{E}[X^2] = \sum_{k=1}^{\infty} k^2(1-p)^{k-1}p = p\left(\sum_{k=1}^{\infty}(k+1)k(1-p)^{k-1} - \sum_{k=1}^{\infty} k(1-p)^{k-1}\right)$

$$= p\left(\sum_{k=2}^{\infty} k(k-1)(1-p)^{k-2} - \sum_{k=1}^{\infty} k(1-p)^{k-1}\right)$$

Since $\sum_{k=1}^{\infty} k x^{k-1} = (1-x)^{-2}$ and $\sum_{k=2}^{\infty} k(k-1)x^{k-2} = 2\cdot(1-x)^{-3}$,

$$\mathbb{E}[X^2] = p\left(\frac{2}{p^3} - \frac{1}{p^2}\right) = p^{-2}(2-p) = 190.$$

3(a) $f_U(x) = \begin{cases} \frac{1}{2} & -1 < x < 1 \\ 0 & \text{o/w} \end{cases}$

$$\mathbb{E}[e^U] = \int_{-1}^{1} e^x \tfrac{1}{2}\,dx = \boxed{\tfrac{1}{2}(e - e^{-1})}$$

(b) $F_X(x) = \mathbb{P}(X \le x) = \mathbb{P}(U^4 \le x) = \begin{cases} 1 & \text{if } x \ge 0 \\ 0 & \text{if } x < 0 \end{cases}$

For $0 \le x < 1$, $\mathbb{P}(U^4 \le x)$

$$= \mathbb{P}(-x^{\frac{1}{4}} \le U \le x^{\frac{1}{4}})$$

$$= \int_{-x^{\frac{1}{4}}}^{x^{\frac{1}{4}}} \tfrac{1}{2}\,du = \underline{x^{\frac{1}{4}}}$$

4 (a) Need $\int_0^{\pi}\int_0^1 c e^y \sin x \; dy\,dx = 1$

$$\int_0^{\pi}\int_0^1 e^y \sin x\,dy\,dx = \left(\int_0^{\pi} \sin x\,dx\right)\left(\int_0^1 e^y\,dy\right)$$

$$= \left(-\cos x \Big|_0^{\pi}\right)\left(e^y\Big|_0^1\right)$$

$$= -(-1-(+1))(e-1) = 2(e-1) \quad \therefore\ \underline{c = \frac{1}{2(e-1)}}$$

(b) $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$

$$= \int_0^1 \frac{1}{2(e-1)} e^y \sin x\,dy$$

$$= \frac{1}{2(e-1)} \sin x \int_0^1 e^y\,dy$$

$$= \underline{\frac{1}{2}\sin x \qquad 0 < x < \pi}.$$

Math 461C13 (Panki Kim)
Dec. 17, 2005
Final (100 points)

Name ______

University ID number ______

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Instructions:

- Your exam contains 9 problems. The entire exam is worth 100 points. The point value of each problem is clearly marked.
- Your exam should contain 11 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- You have 180 minutes for this Final. Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. No calculators are allowed. If in doubt, ask for clarification.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Make sure to do your own work on the exam. Good Luck!

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Problem #1 (10 pts) ______

Problem #2 (10 pts) ______

Problem #3 (10 pts) ______

Problem #4 (10 pts) ______

Problem #5 (10 pts) ______

Problem #6 (10 pts) ______

Problem #7 (10 pts) ______

Problem #8 (20 pts) ______

Problem #9 (10 pts) ______

TOTAL (100 pts) ______

1. (10pts) Let $A$ and $B$ be events such that

$$\mathbf{P}(A) = \frac{1}{3}, \quad \mathbf{P}(A \cup B) = \frac{5}{8} \quad \text{and} \quad \mathbf{P}(A \cap B) = \frac{1}{10}.$$

Find

(a) (5pts) $\mathbf{P}(A \cap B^c)$,

(b) (5pts) $\mathbf{P}(A^c \cup B^c)$.

2. (10pts) The joint density function of $(X, Y)$ is given by

$$f_{X,Y}(x, y) = \begin{cases} c\,(y - x) & \text{if } \; 0 \le x < y \le 1 \\ 0 & \text{otherwise} \end{cases} .$$

(a) (5pts) Find $c$.

(b) (5pts) Find the marginal density function of $X$.

3. (10pts) $X$ is uniformly distributed on $(0, 1)$ and $Y$ is uniformly distributed on $(1, 2)$. We assume that $X$ and $Y$ are independent. Find the density function of $X + Y$.

4. (10pts) Let $X$ and $Y$ be Bernoulli random variables with whose probability of success is $\frac{1}{2}$ and $\frac{1}{3}$ respectively. We assume that the probability of success for $X$ given success of $Y$ is $\frac{1}{4}$.

   Find the covariance of $X$ and $X - Y$.

5. (10pts) Let $X$ denote the point chosen uniformly over the interval $(0, 2)$. Find the distribution function $F_Y$ of $Y$ where

$$Y = \max(X, 1) = \text{maximum between } X \text{ and } 1.$$

Hint: $Y$ is neither discrete nor continuous random variable. $F_Y$ will be a multi-part function. Be careful on the limits of each interval where $F_Y$ is defined.

6. (10pts) Each ball is distributed into one of $8$ boxes equally likely. Suppose $10$ balls are distributed at random into $8$ boxes. We assume that each box is big enough to contain all $10$ balls. Find the expected number of empty boxes.

7. (10pts) $X$ is the angle in radians chosen uniformly from $(-\frac{\pi}{2}, \frac{\pi}{2})$. Let $Y = \sin(X)$. Find the mean and variance of $Y$

$$\text{Hints:} \quad \sin^2(x) = \frac{1-\cos(2x)}{2}, \quad \sin(\frac{\pi}{2}) = \cos(0) = 1$$

$$\sin(-\frac{\pi}{2}) = \cos(\pi) = \cos(-\pi) = -1, \quad \cos(\frac{\pi}{2}) = \cos(-\frac{\pi}{2}) = \sin 0 = \sin(\pi) = \sin(-\pi) = 0$$

8. (20pts) $X$ is a random variable uniformly chosen on $(0, 1)$. Let $Y$ be a random variable uniformly chosen on $(X, 1)$.

   (a) (5pts) Find the joint density function $f_{X,Y}(x, y)$ of $X, Y$.

   (b) (5pts) Find $\mathbf{E}[(1 - X)Y]$.

(c) (5pts) Find the conditional density function $f_{X|Y}(x|y)$ of $X$ given $Y = y$.

(d) (5pts) Find $\mathbf{E}[X|Y = \frac{1}{2}]$.

9. (10pts) Let $X$ and $Y$ be independent, identically distributed random variables whose density $f(x)$ is defined by:

$$f(x) = \begin{cases} c\,e^{-x} & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} .$$

(a) (3pts) Find c.

(b) (7pts) Find the moment generating function of $X - Y$.

# Math 461 C13, Fall, 2005, Final Solution

Note Title 12/16/2005

#1 (a) $\mathbb{P}(A\cap B^c)+\mathbb{P}(A\cap B)=\mathbb{P}(A)$. So $\mathbb{P}(A\cap B^c)=\frac{1}{3}-\frac{1}{10}$

(b) $\mathbb{P}(A^c\cup B^c)=\mathbb{P}((A\cap B)^c)=1-\mathbb{P}(A\cap B)=1-\frac{1}{10}$

#2 (a) $\int_0^1\int_0^y c(y-x)\,dx\,dy=1$

$\int_0^1\int_0^y (y-x)\,dx\,dy=\int_0^1 yx-\frac{1}{2}x^2\Big|_0^y dy=\frac{1}{6}y^3\Big|_0^1=\frac{1}{6}$, $\therefore c=6$

(b) $f_X(x)=\int_0^1 f_{X,Y}(x,y)\,dy=\int_x^1 c(y-x)\,dy=6\left(\frac{1}{2}y^2-xy\right)\Big|_x^1=6\left(\frac{1}{2}(1-x^2)-x(1-x)\right)=3(x-1)^2$

if $0\le x\le 1$ and zero otherwise.

#3 $f_X(x)=\begin{cases}1 & 0<x<1\\ 0 & \text{o/w}\end{cases}$ $f_Y(y)=\begin{cases}1 & 1<y<2\\ 0 & \text{o/w}\end{cases}$, Let $Z=X+Y$

$f_Z(a)=0$ if $a\notin(1,3)$. Assume $1<a<3$

$f_Z(a)=\int_{-\infty}^{\infty} f_X(x)f_Y(a-x)\,dx$. Since $f_Y(a-x)=\begin{cases}1 & a-2<x<a-1\\ 0 & \text{o/w}\end{cases}$

$$f_Z(a)=\begin{cases}\int_0^{a-1}1\,dx=a-1 & \text{if } 1<a<2\\ \int_{a-2}^{1}1\,dx=1-(a-2)=3-a & \text{if } 2\le a<3\end{cases}$$

#4 $\mathbb{P}(X=1)=\frac{1}{2}$, $\mathbb{P}(Y=1)=\frac{1}{3}$, $\mathbb{P}(X=1\mid Y=1)=\frac{1}{4}$

$\mathrm{Cov}(X,X-Y)=\mathrm{Var}(X)-\mathrm{Cov}(X,Y)=\mathbb{E}[X^2]-\mathbb{E}[X]^2-[\mathbb{E}[XY]-\mathbb{E}[X]\cdot\mathbb{E}[Y]]$

$=\mathbb{P}(X=1)-\mathbb{P}(X=1)^2-[\mathbb{P}(Y=1)\mathbb{P}(X=1\mid Y=1)-\mathbb{P}(X=1)\cdot\mathbb{P}(Y=1)]$

$=\frac{1}{2}-\frac{1}{4}-[\frac{1}{3}\cdot\frac{1}{4}-\frac{1}{2}\cdot\frac{1}{3}]=\frac{1}{3}$

#5 $F_Y(a)=0$ if $a<1$, $F_Y(a)=1$ if $a\ge 2$

For $1\le a<2$, $F_Y(a)=\mathbb{P}(\max(X,1)\le a)=\mathbb{P}(X\le a,\ 1\le a)=\mathbb{P}(X\le a)$

$=\frac{a}{2}$

#6 $X_i=\begin{cases}1 & \text{if } i\text{th box is empty}\\ 0 & \text{o/w}\end{cases}$ $X:=\#$ of empty boxes.

$\mathbb{E}[X]=\mathbb{E}[X_1]+\cdots+\mathbb{E}[X_8]=8\cdot\mathbb{P}(X_1=1)=8\cdot\left(\frac{7}{8}\right)^{10}$

$\mathbb{P}(X_1=1)=\left(\frac{7}{8}\right)^{10}$

#7 $f_X(x) = \begin{cases} \frac{1}{\pi} & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & o/w \end{cases}$

$\mathbb{E}[Y] = \int_{-\infty}^{\infty} \sin x \, f_X(x) dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin x) \frac{1}{\pi} dx = \left(-\cos x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\right) \frac{1}{\pi} = 0$

$\mathbb{E}[Y^2] = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\pi} \sin^2 x \, dx = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 - \cos(2x) dx = \frac{1}{2\pi}\left[ x - \frac{1}{2}\sin 2x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\right.$

$= \frac{1}{2\pi}[\pi - 0] = \frac{1}{2} = Var(Y)$

#8 $f_X(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & o/w \end{cases}$ $\quad f_{Y|X}(y|x) = \begin{cases} \frac{1}{1-x} & x < y < 1 \\ 0 & o/w \end{cases}$

(a) $f_{X,Y}(x,y) = f_X(x) \cdot f_{Y|X}(y|x) = \begin{cases} \frac{1}{1-x} & 0 < x < y < 1 \\ 0 & o/w \end{cases}$

(b) $\mathbb{E}[(1-X)Y] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (1-x) y \, f_{X,Y}(x,y) dx dy$

$= \int_0^1 \int_0^y (1-x) y \cdot \frac{1}{1-x} \quad dx\, dy$

$= \int_0^1 y \int_0^y dx\, dy = \frac{1}{3} y^3 \Big|_0^1 = \frac{1}{3}$

(c) $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx = \int_0^y \frac{1}{1-x} dx = -\ln(1-x)\Big|_0^y = -\ln(1-y)$

$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{\frac{1}{1-x}}{-\ln(1-y)} = -\frac{1}{(1-x)\ln(1-y)}$ if $0 < x < y < 1$

and $f_{X|Y}(x|y) = 0 \quad o/w$

(d) $\mathbb{E}[X | Y = \frac{1}{2}] = \int_{-\infty}^{\infty} x f_{X|Y}(x|\frac{1}{2}) dx = \frac{1}{\ln 2} \int_0^{\frac{1}{2}} \frac{x}{1-x} dx = \frac{1}{\ln 2}\left(-\frac{1}{2} + \ln 2\right)$

$\left( \frac{x}{1-x} = \frac{-1+x+1}{1-x} = -1 + \frac{1}{1-x}, \quad \int_0^{\frac{1}{2}} \left(-1 + \frac{1}{1-x}\right) dx = -\frac{1}{2} - \ln(\frac{1}{2}) \right)$

So $\mathbb{E}[X | Y = \frac{1}{2}] = 1 - \frac{1}{2\ln 2}$

#9 (a) $1 = c \int_0^1 e^{-x} dx = (-c) \cdot e^{-x}\Big|_0^1 = (-c)(e^{-1} - 1) \Rightarrow \quad c = \frac{1}{1-e^{-1}} = \frac{e}{e-1}$

(b) $M_{X-Y}(t) = \mathbb{E}[e^{tX - tY}] = \mathbb{E}[e^{tX} e^{-tY}]$

$= \mathbb{E}[e^{tX}] \cdot \mathbb{E}[e^{-tY}] = M_X(t) \cdot M_Y(-t)$

$M_X(t) = M_Y(t) = c\int_0^1 e^{tx} \cdot e^{-x} dx = c \int_0^1 e^{x(t-1)} dx = \frac{c}{t-1} e^{x(t-1)} \Big|_0^1$

$= \frac{c}{t-1}(e^{t-1} - 1) = \frac{e}{(e-1)(t-1)}(e^{t-1} - 1)$

$M_{X-Y}(t) = \left(\frac{e}{e-1}\right)^2 \left( \frac{1}{t-1}(e^{t-1} - 1) \cdot \left(\frac{1}{-t-1}\right)(e^{-t-1} - 1)\right)$